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# Positive Selections of Employees<sup>\*</sup>

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#### Abstract

Utility of working, which captures a worker's idiosyncratic valuation for serving a firm, is an important economic variable that influences a worker's career but has received limited attentions in the literature. This article examines its role in driving labor-market dynamics. I show that a partnership of utility of working—a taste-based, private-value, and time-varying match—and promotion signaling models à la Waldman (1984) rationalizes a wide range of phenomena concerning career dynamics. In particular, the new sequential-screening approach to study turnover provides a micro-foundation for the intra-firm retention policy, which is often taken as a black box in search-and-matching frameworks.

Keywords: utility of working; career dynamics; sequential screening; promotion signaling

# 1 Introduction

Numerous empirical studies suggest that workers often changes jobs *within and/or across* firms. The Longitudinal Employer-Household Dynamics (LEHD) program from the U.S. suggests that worker flows across firms represents approximately 4% of employment and 30% of separations each quarter (Bjelland et al., 2011), and job-to-job transitions account for roughly 50% of all worker reallocations (Haltiwanger, Hyatt, and McEntarfer, 2018). Among workers staffing high-level positions, external recruits account for roughly 20% of all appointments to top management positions (Agrawal, Knoeber, and Tsoulouhas, 2006).

Existing theoretical research has, however, taken two *separate* approaches. One focuses on internal labor markets and highlights the importance of employee motivations and selections inside firms (Gibbons and

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Waldman, 1999a; Lazear and Oyer, 2013). The other approach centers on external labor markets and uses search-and-matching frameworks (Mortensen, 1986; Mortensen and Pissarides, 1999) to capture macro-level turnover dynamics, but typically takes internal structures and human resource management inside firms (such as employee retention and promotion policies) as a black box.

Given the dichotomy of theoretical research, this article takes a step towards *bridging* internal and external labor markets. I show that a wide range of phenomena concerning career dynamics—encompassing internal labor markets, external labor markets, and these two markets' interactions—can be rationalized by a theoretical framework which, as described in Section 2, enriches promotion-signaling models à la Waldman (1984) with a taste-based, private-value, and time-varying match. In particular, the match captures a worker's idiosyncratic utility for serving a firm, a.k.a. utility of working. To fix ideas, consider a worker employed by a firm. Various non-pecuniary aspects of employment can affect this worker's utility of working, e.g., how well he adapts to the organization environment, how much he loves the corporate culture, how well he gets along with the boss and coworkers, etc.<sup>1,2</sup>

#### The model

I am not the first to introduce the idea of utility of working to the literature and model it as a taste-based and private-value match. In two-period frameworks, Acemoglu and Pischke (1998), Schönberg (2007), and Kahn (2013) show that turnover arises in equilibrium, because a worker who drew a low-quality match in the first period intends to improve his match in the second period. However, no prior study has paid attention to the role of utility of working in a worker's long-term career. To this end, I begin with a simple multi-period analysis with a focus on job changes across firms, where in each period the match is only privately revealed to the worker, and with a positive probability the match is resampled. I solve for an employer's retention policy: a (retention) wage schedule. In each period, a worker's mobility decision (i.e., whether to switch firms) satisfies the reservation-match-quality property. That is, in each period a worker separates (resp. stays) with the employer if his match is below (resp. above) that period's reservation match quality. I refer to the wage schedule as a sequential-screening schedule, because it sequentially weeds out workers during a long-term employment relationship.

To incorporate job changes within and/or across firms, I then integrate the sequential-screening anal-

<sup>&</sup>lt;sup>1</sup>According to Linkedin surveys, compensation, work/life balance, impact, culture, career paths, pride, security, challenging, development, colleagues, contributions, and superiors are the top twelve factors when job candidates evaluate different job offers. See https://www.linkedin.com/business/talent/blog/talent-acquisition/ what-recent-college-graduates-are-looking-for-in-first-job.

 $<sup>^{2}</sup>$ Rosen (1974) in his hedonic wage theory considers a labor market where firms offer different working conditions. In equilibrium, workers receive high wage payments as a compensation for poor working conditions. I, however, study a labor market with homogeneous firms, where each worker draws an idiosyncratic taste-based match that can differ across firms.

ysis described above with a promotion-signaling framework à la Waldman (1984), which builds upon the asymmetric-employer-learning hypothesis that a current employer obtains more accurate information about worker ability than prospective employers. The labor market is competitive, where firms are homogeneous and workers differ in ability and schooling levels. Each firm offers two different types of job positions: labor positions and managerial positions, where managerial positions better leverage worker ability than labor positions. In each period, each current employer assigns each worker it employed to either the labor position or the managerial position, and each prospective employer makes poaching offers based on each current employer's job assignments, which signal worker ability given asymmetric employer learning; each current employer can counteroffer; each worker decides whether to switch firms based on his match and the offers he has received. In equilibrium, as detailed in Section 3, during a long-term employees with heterogeneous ability and schooling levels. That is, the retention policy to *positively select* employees with low-quality matches and low ability, while the promotion policy gradually sorts higher-ability workers into higher-level positions based on their schooling levels.

#### Implications

As established in 4, the model captures three broad sets of empirical findings. Regarding internal-labormarket results, the model echoes wage and mobility patterns concerning career dynamics *within* firms. Given asymmetric employer learning, assigning a worker to the managerial position sends a positive signal to prospective employers which raises the poaching wage. In equilibrium, as a worker with a higher schooling level is already considered to be high ability with a higher probability, the wage increase due to a promotion declines with a worker's schooling level; in turn, a worker's promotion prospect increases with his schooling level, i.e., the promotion policy is biased towards more educated workers.

As for external-labor-market results, the model nests wage and mobility patterns concerning career dynamics *across* firms. In particular, the model yields a wage schedule with rent extractions, where a current employer first decreases its retention wage intensively to screen out workers with low-quality matches and, in turn, is able to subsequently decrease its retention wage to exploit rents from retained workers with high-quality matches. This *rent-extraction motive* implies that the current employer undercuts prospective employers when retaining an incumbent worker, which translates to a wage increase for switching firms. Further, as the current employer screens out workers intensively to extract rents in later periods from workers with longer tenure, reservation match qualities decline with tenure. In turn, the resulting turnover dynamics meet with stylized facts that i) the hazard rate of job ending declines with tenure; ii) many new jobs end early but long-term employment relationships are common. The model also speaks to "interaction" results concerning career dynamics *within and across* firms. For the effects of turnover on future careers, due to the aforementioned positive selections of employees, workers employed for a while are, on average, higher ability and more satisfied with current employers than newly employed workers; in turn, workers employed for a while are subsequently more likely to be promoted and less likely to switch firms than newly employed workers. As for the effects of promotions on future careers, due to positive selections of employees again, workers are less likely to change employers after being promoted, because workers promoted are higher ability than workers not promoted; in particular, due to their higher ability and greater satisfactions with current employers than other incumbent employees, workers previously promoted and retained are especially unlikely to separate.

#### Contributions

Utility of working, which captures a worker's idiosyncratic valuation for serving a firm, is clearly an important economic variable that influences a worker's career and, in turn, drives labor-market dynamics. Yet, it has received very limited attentions in the literature. As detailed in Section 5.1, this article contributes to the literature by showing that a partnership of utility of working—a taste-based, private-value, and time-varying match—and promotion signaling models à la Waldman (1984) can rationalize a wide range of phenomena concerning career dynamics within and/or across firms.

As detailed in Section 5.2, this article proposes a novel sequential-screening approach to study turnover, where the taste-based match is different in nature than the productivity-based specific value/match in searchand-matching frameworks. When the match is an idiosyncratic taste-based value, it is natural to be privatevalue. In turn, the current employer's retention policy corresponds to a sequential-screening schedule. In equilibrium, a career-minded worker switches employer *both* because of his intention to improve his match as well as a current employer's rent-extraction motive discussed above. This sets contrast to search-andmatching frameworks where turnover is *solely* driven by a worker's intention to improve his match. In this regard, this article provides a micro-foundation for the intra-firm retention policy which, as pointed out in Lazear and Oyer (2013), is often taken as a black box in search-and-matching frameworks.

This article also contributes to the growing literature of integrative models (Harris and Holmström, 1982; MacLeod and Malcomson, 1988; Gibbons and Waldman, 1999b, 2006; Pastorino, 2015; Ke, Li, and Powell, 2018; Ferreira and Nikolowa, 2023; DeVaro et al., 2024), which attempt to explain a broad set of phenomena rather than a single or a small set of facts. As noted above, the economics of organizations literature highlights the importance of intra-firm employee *selections* and *motivations*. By considering a new aspect of employee selections regarding utility of working, as established in Section 5.3, this article uses selection-based theory to rationalize a wide range of phenomena concerning career dynamics (esp. "interaction" results), which an incentive-based theory does not explain.

# **1.1** Related literature

As noted above, this article connects to two *separate* strands of theoretical studies.

#### External labor markets

There are two workhorse approaches to study turnover dynamics. One is on the basis of search frameworks (Parsons, 1972; Burdett, 1978; Mortensen, 1978; Jovanovic, 1979a) with specific capital—a value that increases over time within the employment relationship but has no value outside the relationship, where each period one party—a worker (resp. a firm)—inspects a random outside value drawn from some distribution, and the worker quits with the firm (resp. the firm layoffs the worker) if the realized outside value is greater than the worker's (resp. the firm's) share of the specific value. Because the specific value increases over time, these models predict that turnover rates decline with tenure. The other builds upon matching frameworks (Jovanovic, 1979b; McLaughlin, 1991; Mortensen and Pissarides, 1994; Moscarini, 2005), where each period's worker output is a noisy public signal of the match, and the match is an experience good whose quality is only learned after an employment spell. These frameworks satisfy a reservation-match-quality property, where each period a relationship is ended if the updated (expected) match is below the reservation match quality. As workers with higher quality matches stay with the firm for a longer time, turnover rates decline with tenure.<sup>3</sup>

In this article, the match is taste-based and private-value, as opposed to search-and-matching frameworks where the specific value or the match is typically productivity-based and public-value. In Section 5, I discuss in detail the role of a taste-based and private-value match in driving career dynamics, and show that the sequential-screening approach to study turnover serves as a micro-foundation for the intra-firm employee retention policy.

#### Internal labor markets

This article is related to the extensive literature building on the asymmetric-employer-learning hypothesis that a current employer obtains more accurate information about worker ability than prospective employers.<sup>4</sup>

<sup>&</sup>lt;sup>3</sup>A third approach is based on Farber (1994), where each worker has an idiosyncratic and fixed separation rate. In equilibrium, workers with longer tenure have lower separation rates and, in turn, are less likely to separate than workers with shorter tenure. This reduced-form approach, though parsimonious, abstracts away from firms' wage determinations and workers' turnover decisions.

<sup>&</sup>lt;sup>4</sup>Other early models building on the hypothesis include Lazear (1986), Milgrom and Oster (1987), Ricart-I-Costa (1988), Waldman (1990), Katz and Ziderman (1990), Bernhardt and Scoones (1993), Laing (1994), Chang and Wang (1996), and Acemoglu and Pischke (1998); more recent theoretical contributions include Ekinci, Kauhanen, and Waldman (2019), Friedrich

In this literature, one class of studies center on job assignments within firms, where a current employer sends a positive signal of a worker's ability to prospective employers when it assigns the worker to a higher-level job position (Waldman, 1984; Gibbs, 1995; Bernhardt, 1995; Zábojník and Bernhardt, 2001; DeVaro and Waldman, 2012). These models, however, do not predict turnover in equilibrium because a current employer can counteroffer to outbid prospective employers whenever the latter are not significantly more productive than the former.<sup>5</sup> The other class of studies abstract away from job assignments within firms but focus on turnover across firms (Greenwald, 1986; Acemoglu and Pischke, 1998; Li, 2013; Ferreira and Nikolowa, 2023), where a current employer sends a negative signal of a worker's ability to prospective employers when it fails to retain the worker. This article integrates these analyses by providing a theoretical framework which captures equilibrium behavior that workers change jobs within and/or across firms.

# 2 The Model

This section presents a simple three-period framework, which enriches promotion-signaling models à la Waldman (1984) with a taste-based, private-value, and time-varying match.

#### The environment

I consider a labor market with free entry, where there are B > 3 homogeneous firms actively competing for workers who differ in ability and schooling levels.

Each period, a continuum of workers—whose careers span for three periods and supply of labor is inelastic and indivisible—enter the labor market. Upon entering the labor market, each worker is endowed with a publicly observable schooling level  $k \in [0, \bar{k}]$  and an initially unknown ability level  $\theta \in \{low, high\}$ , both of which are fixed over time. It is natural that more educated workers have higher expected ability, i.e., the conditional probability  $p(\theta|k)$  satisfies the Monotone Likelihood Ratio Property or  $\frac{\partial}{\partial k} \left(\frac{p(l|k)}{p(h|k)}\right) < 0.^6$ 

Each firm owns a common constant-return-to-scale production technology in which labor is the only input, and offers two different types of job positions where *M*anagerial positions better leverage worker

<sup>(2023),</sup> Ferreira and Nikolowa (2023), DeVaro et al. (2024), and Waldman and Yin (Forthcoming). See Bar-Isaac, Jewitt, and Leaver (2021) for an analysis with symmetric and asymmetric employer learning, and Bar-Isaac and Lévy (2022) and Bar-Isaac and Leaver (2022) for follow-up analyses where asymmetric learning arises endogenously.

<sup>&</sup>lt;sup>5</sup>An exception is DeVaro et al. (2024) where turnover occurs whenever a current employer mistakenly fails to counteroffer. <sup>6</sup>To establish equilibrium uniqueness below, I assume  $\frac{p(l|k=0)}{p(h|k=0)} = \infty$  for the least educated workers and  $\frac{p(l|k=\bar{k})}{p(h|k=k)}$  is sufficiently small for the most educated workers.

ability than Labor positions. In particular, the output of a  $\theta$  ability worker on position J equals

$$y_J(\theta) = \begin{cases} 1 & \text{if } J = L, \\ \alpha\theta & \text{if } J = M, \end{cases}$$

where  $\alpha > 1$  captures the higher importance of ability on managerial positions than on labor positions. Without loss of much generality, I normalize ability levels to l = 0 and h = 1, which yield  $y_M(h) = \alpha > 1 = y_L(h)$  and  $y_M(l) = 0 < 1 = y_L(l)$ , i.e., it is exante efficient to assign a high (resp. low) ability worker to the managerial (resp. labor) position.<sup>7</sup> It is assumed that each worker must staff the labor position before being able to staff the managerial position, then each newcomer to the labor market is initially assigned to the labor position. There is a return to experience that each worker's output is multiplied by  $n_t$ , where  $n_2 > n_1 > 1 = n_0$ , if he has t period(s) of labor-market experience. Also, there is a return to tenure that each worker's output is further multiplied by  $1+\gamma$ , with  $\gamma > 0$ , if the worker stays with the previous period's employer.

One central building block of the model is that during each period, a worker experiences utility of working, i.e., a taste-based match capturing the worker's idiosyncratic valuation for serving a firm, with a time-varying structure that each period the match is possibly resampled. Consider a worker experienced match  $\eta_t^j$  at firm j in period t. At the beginning of period t + 1, the worker updates the match with firm j: with probability  $q \in (0,1)$  he retains the match, i.e.,  $\eta_{t+1}^j = \eta_t^j$ , and with probability 1 - q he samples a new match, i.e.,  $\eta_{t+1}^j \stackrel{d}{\sim} G(\cdot)$ . If the worker switches to a different firm  $j' \neq j$  in period t+1, he always samples a new match, i.e.,  $\eta_{t+1}^{j'} \stackrel{d}{\sim} G(\cdot)$ , but he has to be employed by the new firm to discover the match quality. It is natural that the taste-based match is privately revealed to each worker, and independently distributed with each worker's ability and schooling level. Without loss of generality,  $G(\cdot)$  is log-concave and has a zero mean.<sup>8</sup>

Another central building block of the model is asymmetric employer learning concerning worker ability. That is, each worker's actual ability is initially unknown when he enters the labor market. But during employment each worker's ability is privately revealed to his current employer (and to himself), but not to any prospective employer.

To close the model, all players are rational, risk-neutral, and share a common discount factor  $\delta \in (0, 1]$ . Each worker incurs no cost to switch firms. As assumed in Bernhardt (1995) and Tirole (2016), exits are

<sup>&</sup>lt;sup>7</sup>I abstract away from positional constraints, i.e., each firm can freely assign a worker to either position. See Ke, Li, and Powell (2018), Friedrich (2023), and Waldman and Yin (Forthcoming) for related analyses with positional constraints.

<sup>&</sup>lt;sup>8</sup>To ease the exposition, the match is assumed to be the same across job positions within a firm. In the real world, it is natural that each worker appreciates the status as a manager due to the relative place in the hierarchy inside a firm (Frank, 1984), e.g., utility is  $\eta$  as a labor worker and  $\eta + z$  as a manager where an exogenous parameter z > 0 describes incremental utility each worker derives from the status as a manager. Given a competitive market with homogeneous firms, the status value z > 0 will, however, not change equilibrium results described below.

$\theta \in \{l,h\}$	ability level: low or high
$k\in [0,\bar{k}]$	schooling level
$y_J( heta)$	a $\theta$ ability worker's output on position $J \in \{Labor, Manager\}$
$\alpha > 1$	importance of worker ability on managerial positions
$n_t$	multiplier for the return to t periods of experience, with $n_2 > n_1 > 1 = n_0$
$1 + \gamma$	multiplier for the return to tenure, with $\gamma > 0$
$q \in (0,1)$	probability of retaining the previous period's match

Table 1: Notations.

absorbing, i.e., a worker cannot return to an employer after quitting.<sup>9</sup> The per-period payoff is equal to "wage plus utility of working" for a worker, and "output minus wage" for a firm. In cases requiring tie-breaking, a worker switches firms whenever it yields a strictly higher payoff than staying with the current employer. Similarly, a worker is promoted if the current employer derives a strictly higher profit from assigning the worker to the managerial position than to the labor position.

## The game

Following promotion-signaling models à la Waldman (1984), I focus on spot contracting for the wagedetermination process, i.e., at the beginning of each period (i.e., prior to productions) firms extend wage offers to workers.<sup>10</sup> The sequence of moves for each period t is as follows. At the beginning of period t, each current employer makes a job assignment for each worker it employed during period t - 1; for each worker, each prospective employer observes the current employer's job assignment and makes a poaching offer, which is followed by the current employer's counteroffer;<sup>11</sup> each worker then updates the match for period t with the current employer, and chooses whether to switch firms. During period t, each worker conducts production and experiences the match, while each employer observes its worker's ability.

I focus on perfect Bayesian equilibrium. That is, player strategies and beliefs satisfy: i) consistency: based on observed actions (i.e., whether and when a worker has changed jobs within and/or across firms) and individual characteristics (i.e., a worker's schooling level and labor-market experience), each prospective employer adjusts belief concerning the worker's ability, which is consistent with Bayes' rule on equilibrium paths (beliefs can be arbitrary off equilibrium paths); and ii) sequential rationality: given prospective employers' beliefs, each current employer's job-assignment and retention decisions as well as each worker's choice of job offers are optimal in expectation.

Define  $\phi(r) = \max_{\eta} (1 - G(\eta)) (r + \eta)$ . To rule out uninteresting cases where all high-ability workers are

<sup>&</sup>lt;sup>9</sup>See Yin and Zax (2024) for a related analysis of boomerang workers.

<sup>&</sup>lt;sup>10</sup>As detailed in the Web Appendix, long-term or dynamic contracting is time-inconsistent given a time-varying match.

<sup>&</sup>lt;sup>11</sup>A bidding process with counteroffers is first assumed in Greenwald (1986) and Milgrom and Oster (1987), and Barron, Berger, and Black (2006) present evidence on the common use of counteroffers.

assigned to managerial positions at the same time, I assume

$$\gamma n_1(\alpha - 1) < \delta \left[ \phi(\gamma n_2 \alpha) - \phi(\gamma n_2) \right],\tag{A1}$$

which holds if  $n_2$  is sufficiently larger than  $n_1$  for the return to experience. In turn, the model yields career dynamics where high-ability workers are gradually sorted into managerial positions.

# 3 The Analysis

I first study a simplified model with a focus on employee retention inside a firm. I then analyze the full model, which is shown in Section 4 to capture a wide range of phenomena concerning career dynamics.

# 3.1 The simplified analysis

In the simplified model described below that focuses on a current employer's retention policy, I abstract away from two aspects of the full model: the current employer's promotion policy and prospective employers' poaching.

#### The simplified model

Consider a worker employed for  $\tau$  periods in period t. At the beginning of period t, the worker receives a retention offer  $w_t^{\tau}$  from his current employer; the worker updates the match for period t with the current employer, i.e., with probability  $q \in (0, 1)$  he retains the previous period's match, and with probability 1 - q he samples a new match from  $G(\cdot)$ . If the worker stays in period t, he receives wage  $w_t^{\tau}$ , produces output  $y_t^{\tau}$ , and experiences the match during period t; if he exits in period t, he obtains the payoff to exit  $e_t^{\tau}$ , and the game ends.<sup>12</sup>

#### The current employer's retention policy: a (retention) wage schedule

I now solve for the current employer's retention policy: a (retention) wage schedule. In each period, a worker's mobility decision (i.e., whether to switch firms) satisfies the reservation-match-quality property. That is, as the current employer cannot observe a worker's match quality, each worker employed for  $\tau$  periods receives a common retention offer  $w_t^{\tau}$  in period t, and quits (resp. stays) if the updated match is below (resp. above) the reservation match quality  $\hat{\eta}_t^{\tau}$ . Then, the (retention) wage schedule corresponds to a *sequential-screening* schedule, which sequentially weeds out workers during a long-term relationship.

 $<sup>^{12}</sup>y_t^{\tau}$  and  $e_t^{\tau}$  are exogenous in the simplified analysis but endogenously determined (by the current employer's promotion policy and prospective employers' poaching) in the full analysis.



Figure 1: Two possible regimes for the retention policy.

#### Sequential screening

Call the three periods of a worker's career periods 0, 1, and 2. As detailed below, for a worker employed in period 1, I derive the reservation match quality for period 2, i.e.,  $\hat{\eta}_2^1$ ; for a worker employed in period 0, I pin down two different reservation match qualities, i.e.,  $\hat{\eta}_1^1$  for period 1 and  $\hat{\eta}_2^2$  for period 2, which as discussed above correspond to a sequential-screening schedule with two stages of screening.

One-stage screening for a worker employed in period 1. Intuitively, a worker's output  $y_2^1$  represents the current employer's return to retaining him, while the payoff to exit  $e_2^1$  serves as a proxy for the current employer's retention cost. When the value of "output less payoff to exit"  $y_2^1 - e_2^1$  increases, the current employer responds by raising its retention offer, which results in a decreased reservation match quality  $\hat{\eta}_2^1$ .

Sequential screening for a worker employed in period 0. When the worker almost never retains the period 1 match in period 2, i.e.,  $q \rightarrow 0$ , there is no linkage of the period 1 match and the period 2 match. In turn, the period 1 reservation match quality  $\hat{\eta}_1^1$  has no impact on the period 2 reservation match quality  $\hat{\eta}_2^2$ , which is equal to  $\hat{\eta}^*$  and referred to as the *benchmark* quality. When the worker retains the period 1 match in period 2 with probability  $q \in (0, 1)$ , there is an inter-temporal connection between the period 1 match and the period 1 match. As depicted in Figure 1, there are two possible regimes.

- 1. If  $\hat{\eta}_1^1 \leq \hat{\eta}^*$ , the period 2 reservation match quality coincides with the benchmark quality, i.e.,  $\hat{\eta}_2^2 = \hat{\eta}^*$ .
- 2. If  $\hat{\eta}_1^1 > \hat{\eta}^*$ , the period 2 reservation match quality exceeds the benchmark quality, i.e.,  $\hat{\eta}_2^2 > \hat{\eta}^*$ .

The logic is as follows. Given a worker who stayed in period 1, the current employer realizes that his period 1 match is above  $\hat{\eta}_1^1$ . When  $\hat{\eta}_1^1 > \hat{\eta}^*$ , i.e., the period 1 reservation match quality is sufficiently high, there is an opportunity for price discrimination in period 2, where the current employer decreases the wage offer but still possibly retains the worker. This results in a period 2 reservation match quality higher than the benchmark quality, i.e.,  $\hat{\eta}_2^2 > \hat{\eta}^*$ . In contrast, when  $\hat{\eta}_1^1 \leq \hat{\eta}^*$ , there is no opportunity for price discrimination in period 2, where the benchmark quality, i.e.,  $\hat{\eta}_2^2 > \hat{\eta}^*$ . In contrast, when  $\hat{\eta}_1^1 \leq \hat{\eta}^*$ , there is no opportunity for price discrimination in period 2, where the period 2 reservation match quality, i.e.,  $\hat{\eta}_2^2 = \hat{\eta}^*$ .<sup>13</sup>

<sup>&</sup>lt;sup>13</sup>In terms of the logic, the retention policy with price discrimination is analogous to behavior-based price discrimination (BBPD) in dynamic pricing of a product. The difference is that I apply Tirole's idea of absorbing exits to study a current em-

#### Auxiliary results

As q increases, the current employer knows that with a higher probability a worker retains the previous period's match. In turn, there will be more opportunities for price discrimination, meaning that the period 2 reservation match quality  $\hat{\eta}_2^2$  increases in q. In particular, if  $q \to 1$ , the period 2 reservation match quality coincides with the period 1 reservation match quality, i.e.,  $\hat{\eta}_2^2 = \hat{\eta}_1^1$ . The logic is that if the period 1 match is always retained in period 2, the current employer should screen out workers as early as possible (i.e., just conduct the first stage of screening in period 1), which means that workers quit in period 1 only.<sup>14</sup>

Claim 1. If  $\hat{\eta}_1^1 > \hat{\eta}^*$ , the period 2 reservation match quality satisfies  $\partial \hat{\eta}_2^2 / \partial q > 0$ , with  $\lim_{q \to 1} \hat{\eta}_2^2 = \hat{\eta}_1^1$ .

Given that the period 1 reservation match quality is higher in Regime 2 than in Regime 1 (i.e.,  $\hat{\eta}_1^1 \leq \hat{\eta}^*$ in Regime 1 and  $\hat{\eta}_1^1 > \hat{\eta}^*$  in Regime 2), the retention policy in Regime 2 weeds out workers more intensively than that in Regime 1. Thus, I refer to the retention policy in Regime 2 as *intensive screening* and that in Regime 1 as *non-intensive screening*. In particular, due to price discrimination discussed above, intensive screening is a wage schedule which extracts rents in a long-term relationship.

Claim 2. Intensive screening is a retention policy with rent extractions, where the current employer decreases its period 1 retention offer intensively to screen out employees with matches below  $\hat{\eta}_1^1$  where  $\hat{\eta}_1^1 > \hat{\eta}^*$ , followed by a decrease of its period 2 retention offer to exploit rents from retained employees with matches above  $\hat{\eta}_1^1$ .

Despite extracting rents in a long-term relationship, intensive screening weeds out workers intensively, which entails output losses for exited workers who could otherwise be retained by non-intensive screening. In equilibrium, the retention policy optimally trades off rent extractions and output losses.

Claim 3. Intensive screening is the equilibrium retention policy whenever the value of "output less payoff to exit" in period 1 is less than that in period 2, i.e.,  $y_1^1 + \delta(1-q)\phi\left(y_2^2 - e_2^2\right) - \left[e_1^1 - \delta e_2^2 - \delta(1-q)\varphi\left(\hat{\eta}_2^2\right)\right] < y_2^2 - e_2^2$ , where  $\varphi\left(\hat{\eta}_2^2\right) = \int_{\eta \ge \hat{\eta}_2^2} \eta - \hat{\eta}_2^2 dG(\eta)$ .<sup>15</sup>

# 3.2 The full analysis

This section presents the full analysis, which now incorporates the current employer's promotion policy and prospective employers' poaching.

ployer's retention policy, where each worker cannot come back in the future once he has terminated the employment relationship with the current employer. See the Web Appendix for details.

 $<sup>^{14}</sup>$ Put it different, there is a scope for the second stage of screening (which occurs in period 2) only if the period 1 match is possibly resampled in period 2.

<sup>&</sup>lt;sup>15</sup>Given intensive screening, if a worker separates in period 1, the output loss equals  $y_1^1 + \delta(1-q)\phi(y_2^2 - e_2^2)$  where  $\delta(1-q)\phi(y_2^2 - e_2^2)$  is the period 2 profit that the current employer forgoes, while the payoff to exit equals  $e_1^1 - \delta\left[e_2^2 + (1-q)\varphi(\hat{\eta}_2^2)\right]$  where  $\delta\left[e_2^2 + (1-q)\varphi(\hat{\eta}_2^2)\right]$  is the period 2 payoff to exit that the worker forgoes.

#### Preliminaries

Given asymmetric employer learning, each current employer's job assignments not only sort workers with heterogeneous ability and schooling levels to different job positions, but also serve to signal worker ability to prospective employers. In particular, as assigning a worker to the managerial position sends a positive signal to prospective employers which raises the poaching wage, a current employer chooses to promote a worker only if the associated output increase outweighs the (poaching) wage increase.<sup>16</sup>

As a low ability worker is more productive as a labor worker than as a manager, I find that

#### Lemma 1. In each period, each low ability worker is assigned to the labor position.

I can then focus on job assignments for high ability workers. In period 2, whether a worker has stayed or separated in period 1, assigning a high ability worker to the managerial position increases his output by  $(1 + \gamma)n_2 [y_M(h) - y_L(h)] = (1 + \gamma)n_2(\alpha - 1)$  which outweighs the poaching wage increase  $n_2(\alpha - 1)$ , where the poaching wage is  $n_2\alpha$  to a manager and  $n_2$  to a labor worker when job assignments are fully efficient. In turn, I obtain that

#### Lemma 2. In period 2, each high ability worker is assigned to the managerial position.

Put together, Lemma 1 and 2 suggest a fully efficient promotion policy in period 2. In period 1, assigning a high ability worker to the managerial position increases his output by  $(1 + \gamma)n_1 [y_M(h) - y_L(h)] = (1 + \gamma)n_1(\alpha - 1)$ , which does not outweigh the poaching wage increase  $n_1\alpha + \delta\phi(\gamma n_2\alpha) - (n_1 + \delta\phi(\gamma n_2))$  given (A1), where the poaching wage is  $n_1\alpha + \delta\phi(\gamma n_2\alpha)$  to a manager and  $n_1 + \delta\phi(\gamma n_2)$  to a labor worker when job assignments are fully efficient. In turn, the promotion policy is not fully efficient in period 1, where some high ability workers are assigned to labor positions.<sup>17</sup> To characterize this inefficient promotion policy, I use  $f(\theta; k) \in [0, 1)$  to describe the probability a  $\theta$  ability worker with schooling level k is assigned to the managerial position. I find that

<sup>&</sup>lt;sup>16</sup>It is noteworthy that, as a worker's utility of working is independently distributed with his ability and schooling level, the current employer's job assignments should not depend on whether a worker has previously stayed or separated (although the current employer's profit can be higher for a stayed worker than for a separated worker due to rent extractions described in Claim 2 above).

<sup>&</sup>lt;sup>17</sup>Unlike typical promotion signaling models (Waldman, 1984; Gibbs, 1995; Bernhardt, 1995; Zábojník and Bernhardt, 2001; DeVaro and Waldman, 2012), although promotions reveal a current employer's advantageous information concerning worker ability to prospective employers, asymmetric employer learning by itself does not generate any promotion distortion in this article. That is, in the absence of a taste-based match, job assignments will be fully efficient in each period because the output increase associated with a promotion always exceeds the poaching wage increase. However, with the presence of a taste-based match, a current employer captures profits in period 2, i.e.,  $\phi(\gamma n_2 \alpha)$  from a high ability worker and  $\phi(\gamma n_2)$  from a low ability worker, which result in a higher level of wage increase in period 1. In turn, the output increase is not high enough to warrant a fully efficient promotion policy in period 1.



Figure 2: Period 1 poaching wages.

When  $f(h; k) \in (0, 1)$  for  $\forall k > \hat{k}$  (i.e., a mixed strategy), a current employer is indifferent between assigning the worker to the managerial position and to the labor position, so the wage increase equals the output increase; in contrast, when f(h; k) = 0 for  $\forall k \leq \hat{k}$ , the wage increase outweighs the output increase.

**Lemma 3.** In period 1, there exists a unique threshold schooling level  $\hat{k} \in (0, \bar{k})$  such that a high ability worker can be assigned to the managerial position whenever his schooling level exceeds the threshold, i.e.,  $f(h;k) \in (0,1)$  if  $k > \hat{k}$  and f(h;k) = 0 otherwise.

To pin down the promotion policy  $f(h; k) \in (0, 1)$  for high ability workers with schooling levels above the threshold, consider two workers with different schooling levels but both assigned to labor positions. As depicted in Figure 2, these two workers, though differ in schooling levels, receive an equal poaching wage. Thus, as a worker's schooling level is positively related to his ability, it must be the case that a worker's promotion prospect increases with his schooling level. The logic is that if a worker has a better promotion prospect but is not promoted, then the worker is thought to be low ability with a higher probability.

**Lemma 4.** In period 1, the promotion prospect of a high ability worker (with schooling level above the threshold) increases with his schooling level, i.e.,  $\partial f(h;k)/\partial k > 0$  for  $k > \hat{k}$ .

#### The equilibrium

I now summarize equilibrium behavior with a focus on a current employer's promotion and retention policy.

**Proposition 1.** There exists a unique equilibrium, where a current employer's promotion policy satisfies Lemma 1 through 4 above, and a current employer's retention policy corresponds to a wage schedule with rent extractions (i.e., intensive screening).

The above result suggests *improving* promotional efficiency, i.e., a current employer's promotion policy becomes more efficient for more experienced workers. As a consequence, a current employer's retention policy corresponds to a wage schedule with rent extractions (i.e., intensive screening). Recall from Claim 3 that intensive screening is the equilibrium regime whenever the value of "output less payoff to exit" in period 1 is less than that in period 2. In the full model, a competitive market suggests that prospective employers compete away all future profits when poaching in period 1, which indicates a high value for the payoff to exit in period 1. As for the output value, it rises over time due to improving promotional efficiency (which suggests that each high-ability worker becomes more productive once appointed as a manager) and the return to experience (which entails that each worker gets more productive in period 2 than in period 1).

As a current employer's job assignments serve as signals of worker ability, improving promotional efficiency described above indicates *reducing* information asymmetry for more experienced workers. In particular, due to information asymmetry in period 1, job switchers are *adversely selected* in terms of ability (Gibbons and Katz, 1991; Acemoglu and Pischke, 1998). That is, holding schooling levels, labor-market experience, and job assignments fixed, newly employed workers are on average higher ability than incumbents. Consider a high ability worker and a low ability worker with equivalent schooling levels, labor-market experience, and job assignments. Due to asymmetric employer learning, they should receive an equal poaching wage from prospective employers, whereas a current employer can use its advantageous information to offer ability-contingent retention wages. So, the low ability one receives a lower retention wage and is thus more likely to switch firms than the high ability one. In contrast, the fully efficient promotion policy completely reveals worker ability to prospective employers in period 2, where there is no adverse selection.

The above result also suggests that during a long-term employment relationship, the current employer uses its promotion and retention policy to *positively select* employees with heterogeneous ability and schooling levels. That is, the retention policy sequentially weeds out workers with low-quality matches and low ability, while the promotion policy gradually sorts higher-ability workers into higher-level positions based on their schooling levels.

# 4 Rationalize Empirical Findings

In this section, I demonstrate that the model captures three broad sets of empirical findings.

## 4.1 Internal-labor-market results

To begin with, I consider internal-labor-market results concerning career dynamics within firms.

In line with documented facts that educated workers are favored in the promotion process but this favoritism effect declines with worker age (Baker, Gibbs, and Holmström, 1994a,b; Lluis, 2005; DeVaro and Waldman, 2012; Bognanno and Melero, 2016), I find that

**Corollary 1.** Holding labor-market experience, firm tenure, and job assignments fixed, a worker's promotion prospect increases with his schooling level in period 1, but not in period 2.

Consistent with empirical findings concerning the intra-firm wage gap across job positions (Baker, Gibbs, and Holmström, 1994a,b; McCue, 1996; DeVaro and Waldman, 2012), I also find that

**Corollary 2.** Holding labor-market experience, firm tenure, and job assignments fixed, the wage increase given a promotion decreases with a worker's schooling level in period 1, but not in period 2.

As discussed in Section 3, these two results follow from asymmetric employer learning. In period 1, as schooling levels and job assignments both signal worker ability to prospective employers, the poaching wage to a labor worker is higher given a higher schooling level. So, as depicted in Figure 2, the signal associated with a promotion increases wage less given a higher schooling level; in turn, firms distort promotions less given a higher schooling level, i.e., a worker's promotion prospect increase with his schooling level. By contrast, due to fully efficient job assignments in period 2, a worker's promotion prospect and the wage increase are both independent of his schooling level.

In short, this article enriches promotion signaling models à la Waldman (1984) with a match component. The equilibrium nests standard results described above because, as hinted in footnote 16, a worker's utility of working is independently distributed with his ability and schooling level and, in turn, has no impact on the promotion policy.<sup>18</sup> As detailed in Section 5.1, the new aspect of the equilibrium is that turnover now arises as part of equilibrium behavior. In turn, the model further rationalizes findings described below.

## 4.2 External-labor-market results

I then consider external-labor-market results concerning career dynamics across firms.

Farber (1999), in a survey article, points out that a theory of worker mobility should explain two core facts with strong empirical support: i) the hazard rate of job ending declines with tenure; and ii) many new jobs end early but long-term employment relationships are common. Consistent with these facts, the model generates exit dynamics as follows.

**Corollary 3.** For each worker employed in period 0, the separation rate is higher in period 1 than in period 2, i.e., the hazard rate of job ending declines with tenure. There exist workers who always or never change employers in periods 1 and 2.

This result follows from intensive screening. As described in Claim 1 and 2, to extract rents from incumbent employees with high-quality matches, a current employer chooses to weed out workers intensively

 $<sup>^{18}</sup>$ See Section 5.3 for a setting where utility of working impacts on the promotion policy.

in period 1, which entails a reservation match quality higher in period 1 than in period 2. In turn, the separation rate is higher in period 1 than in period 2.

The model also matches well-documented facts that workers receive a wage increase for switching firms (Bartel and Borjas, 1981; Mincer, 1986; Topel and Ward, 1992; Buchinsky et al., 2010).

**Corollary 4.** In each period, each worker receives a wage increase for switching firms.

This result is also driven by intensive screening. As described in Claim 1 and 2, the current employer decreases its retention wage in period 1 intensively to weed out employees with low-quality matches and decrease its retention wage in period 2 to exploit rents from retained employees with high-quality matches. This rent-extraction motive indicates equilibrium behavior that in each period the current employer undercuts prospective employers when retaining an incumbent worker, which indicates a wage increase for switching firms.

Overall, this article proposes a novel sequential-screening approach to study turnover, which nests standard results described above. Note, in particular, that the taste-based match reflects a worker's utility of working in this article, which is different in nature than the productivity-based specific value or match in search-and-matching frameworks. As detailed in Section 5.1, the result described above concerning the wage change for switching firms will, however, be overturned if the match is productivity-based. Also, as detailed in Section 5.2, sequential-screening provides a micro-foundation for the intra-firm retention policy, because exit dynamics described above are *both* driven by a worker's intention to improve his match as well as a current employer's rent-extraction motive, as opposed to search-and-matching frameworks where turnover is *solely* driven by a worker's intention to improve his match.

## 4.3 "Interaction" results

I now explore "interaction" results concerning career dynamics within and across firms.

For the effect of separations on future careers, consistent with empirical findings (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Bidwell, 2011; Van der Klaauw and Da Silva, 2011; Kauhanen and Napari, 2012) that workers employed for a long time period are more likely to be later promoted and less likely to subsequently separate than workers employed for a short time period, I find that

**Corollary 5.** In period 2, holding schooling levels, labor-market experience and job assignments fixed, workers retained in period 1 have better promotion prospects and lower separation rates than workers separated in period 1.

The results follows from the aforementioned positive selection of employees. That is, workers retained in period 1 are, on average, higher ability and more satisfied with current employers than workers separated in period 1; in turn, workers retained in period 1 are subsequently more likely to be promoted and less likely to switch firms than workers separated in period 1.

As for the impact of promotions on future careers, I obtain results in line with empirical evidences that workers promoted are less likely to switch employers than workers not promoted (Baker, Gibbs, and Holmström, 1994a; Treble et al., 2001; Bidwell, 2011; Van der Klaauw and Da Silva, 2011; Kauhanen and Napari, 2012), and a "positive retention" effect (Benson and Rissing, 2020)—if workers assigned to the managerial (resp. labor) position are taken as high (resp. low) performers, then high performers are less likely to change firms than low performers, with high performers previously retained especially unlikely to switch employers.

**Corollary 6.** In period 2, holding schooling levels, labor-market experience, and firm tenure fixed, workers promoted in period 1 are less likely to switch firms than workers not promoted in period 1; in particular, workers promoted and retained in period 1 are least likely to separate among all incumbents.

The result follows because workers promoted in period 1 must be high ability while workers not promoted in period 1 consist of low ability and high ability ones, so the former are less likely to switch employers than the latter. In particular, managers retained in period 1 are positively selected in terms of ability and matches and, in turn, least inclined to switch firms in period 2 than other incumbents, including labor workers retained in period 1, labor workers newly employed in period 1, and managers newly employed in period 1.

As noted in the Introduction, the economics of organizations literature highlights the importance of intra-firm employee selections and motivations (Gibbons and Waldman, 1999a; Lazear and Oyer, 2013). By considering a new aspect of employee selections regarding utility of working, as discussed in Section 5.3, this article provides a selection-based theory which captures a wide range of phenomena concerning career dynamics (esp. "interaction" results) that an incentive-based theory does not explain.

# 5 Discussions

To better understand utility of working, I elaborate on the role of a taste-based match in driving career dynamics. I also show that sequential screening, as a new approach to study turnover, serves as a microfoundation for the intra-firm employee retention policy.

## 5.1 The role of a taste-based match

To begin with, I elaborate on the role of utility of working, as a taste-based, private-value, and time-varying match, in driving career dynamics.

As hinted in Section 1.1, in typical promotion signaling models à la Waldman (1984) where a current employer can counteroffer, prospective employers cannot hire away a worker whenever the current employer can outbid prospective employers. Consequently, there is a winner's curse for hiring away an external worker, which forces the poaching offer to equal the lowest possible value for all workers with an identical signal. As established in Section 4, this article enriches promotion signaling models with a taste-based match, where turnover arises as part of equilibrium behavior. The logic is that each worker's intention to improve his match shuts down the channel for the winner's curse, because even the highest-ability worker can switch firms if he intends to improve his low-quality match.

As also hinted in Section 1.1, the taste-based match in this article is different in nature than the productivity-based specific value or match in typical search-and-matching frameworks. Recall from Section 4 that, due to the rent-extraction motive, in each period the current employer undercuts prospective employers when retaining an incumbent worker, which indicates a wage increase for switching firms. This result will, however, be overturned if the match is productivity-based à la Jovanovic (1979b). That is, there will be a wage decrease for switching firms because workers employed for a while are more productive ones (associated with higher-quality matches) and, in turn, receive a higher wage than newly employed workers.<sup>19</sup>

When the match is taste-based, it is natural that its quality is an idiosyncratic private value which is resampled with a positive probability in each period. This time-varying structure is essential for generating turnover dynamics described in Section 4. That is, if the match is, instead, time-invariant, i.e., q = 1, Claim 1 above suggests that the current employer should screen workers as early as possible (i.e., just conduct the first stage of screening in period 1); in turn, workers quit in period 1 only. Using this logic, if a worker's career spans for more than three periods, screening will also take place in period 1 only; in turn, exit dynamics are that some workers quit in period 1 but no worker separates in period 2 and onwards, indicating that the hazard rate of job ending does not decline with tenure.<sup>20</sup>

# 5.2 The intra-firm employee retention policy

Continued with the above discussion, I now establish that sequential screening, as a new approach to study turnover, serves as a micro-foundation for the intra-firm employee retention policy.<sup>21</sup>

<sup>&</sup>lt;sup>19</sup>The impact of job mobility on earnings is also the subject of many studies employing search frameworks. In Postel-Vinay and Robin (2002) and Bagger et al. (2014), firms that differ in productivity can renegotiate wages with employees receiving more attractive poaching offers. As more productive firms can better retain incumbent workers and poach new workers, wages should increase for changing jobs across firms. In contrast, this article is based on a setting with homogeneous firms, where a current employer's rent-extraction motive drives the wage increase for switching firms.

<sup>&</sup>lt;sup>20</sup>See the Web Appendix for details.

<sup>&</sup>lt;sup>21</sup>The discussion below centers on a current employer's selections concerning utility of working when retaining an incumbent worker. In terms of worker ability, analogous to internal-labor-market models discussed in Section 1.1 that focus on job separations (Greenwald, 1986; Acemoglu and Pischke, 1998; Li, 2013; Ferreira and Nikolowa, 2023), the current employer's

#### The current employer's rent-extraction motive

Search-and-matching frameworks typically assume a fixed sharing rule, i.e., in each period a worker (resp. firm) receives  $\lambda \in [0, 1]$  (resp. the remainder) of the output where  $\lambda$  is either exogenously given or determined at the beginning of an employment relationship. Take matching frameworks à la Jovanovic (1979b) where the match is productivity-based and public-value for instance, in each period the current employer just offers a wage that equals a fixed fraction of a worker's output. In turn, turnover is *solely* attributed to a worker's intention to improve his match.

By contrast, in this article where the match is taste-based and private-value, a current employer's retention policy corresponds to a sequential-screening schedule, where the current employer strategically chooses the retention wage to extract rents from incumbent workers. This results in an endogenously determined and time-varying sharing rule  $\lambda_t$ , where turnover is driven by *both* a worker's intention to improve his match as well as a current employer's rent-extraction motive. In this regard, sequential screening, as a new approach to study turnover, provides a micro-foundation for the intra-firm employee retention policy, which, as pointed out in Lazear and Oyer (2013), is often taken as a black box in search-and-matching frameworks.

## Supporting evidences

Apart from aforementioned macro-level facts in Section 4 on turnover patterns and wage increases for switching firms, below I find two micro-level facts, which suggest: 1) the importance of utility of working; and 2) the existence of a current employer's rent-extraction motive when retaining incumbent employees.

Based on a firm's personnel data which includes leaving-interview surveys of all separated workers, Benson and Rissing (2020) use employees' self-reported *primary* reasons for quitting to unpack separation reasons. They find that workers who switched firms after certain periods of tenure report that turnover is not largely due to low-quality matches at the current firm but often cite exogenous reasons, whereas workers employed for a short time period quit mainly quoting match-quality reasons. As shown below, when  $q \in (0.5, 1)$ , intensive screening yields exit dynamics compatible with these facts.

**Proposition 2.** In period 2, if  $q \in (0.5, 1)$ , workers employed for one period change employers mainly for match-quality reasons, whereas workers employed for two periods switch firms primarily for exogenous reasons.

Recall that with a positive probability each worker encounters a random shock in each period, where the value of the shock can be instantaneously inspected (as opposed to the match with a new employer which is an experience good in nature). Thus, each period a worker can quit for two types of reasons: i)

retention wage is ability-contingent.

match-quality reasons: though not encountering a shock, the worker quits because he retains the previous period's low-quality match; and ii) exogenous reasons: the worker separates since he has encountered an adverse shock, i.e., he draws a low value for the upcoming period.

In equilibrium, a worker employed for one period can separate for either match-quality reasons (i.e., he retains the previous period's low-quality match), or for exogenous reasons (i.e., he encounters an adverse shock), but  $q \in (0.5, 1)$  suggests a small probability of encountering a random shock in each period. In turn, a newly employed worker separates mainly quoting match-quality reasons. By contrast, a worker employed for two periods is positively selected in terms of his match and thus should not quit whenever he retains the previous period's high-quality match, so he quits mainly citing exogenous reasons.<sup>22</sup>

One might think that a matching framework à la Jovanovic (1979b) can also capture the above fact in Benson and Rissing (2020) if a worker encounters a random shock in each period with a small probability. But in that case, as the match is purely productivity-based and public-value, there will be no point in carrying out leaving interviews and surveying all separated workers, which are costly and time-consuming, from the standpoint of a current employer who can observe each separated worker's performance. Further, Benson and Rissing (2020) provide evidences for both taste-based matches and productivity-based matches in driving turnover. That is, excluding involuntary turnover due to plant closing and position abolishment, documented match-quality reasons include taste-related dissatisfactions (with hours, location, environment, etc.) and productivity-related dissatisfactions (with pay and advancement opportunity), and exogenous reasons consist of retirement, policy violations, other reasons, and declined to say. Hence, facts in Benson and Rissing (2020) suggest that turnover is at least partially driven by a worker's intention to improve his taste-based match, which entails the importance of utility of working.

Another documented fact from Benson and Rissing (2020), which is arguably a direct evidence for one of this article's main insight that a current employer is strategic with a rent-extraction motive, is that "employers select relatively strong internal candidates who are likely to stay because employers consider *perceived loyalty* when they hire internally, which could *extend their rents* if internal hires perform well." Corresponding to the current employer's rent-extraction strategy when retaining incumbent employees in this article, "perceived loyalty" can be interpreted as equilibrium behavior in period 1 that the current employer offers a decreased retention wage (i.e., price discrimination) but an incumbent employee still does not quit, while "extend their rents" meet with equilibrium behavior in period 2 that the current employer exploits rents through decreasing the retention wage to a retained employee.

 $<sup>^{22}</sup>$ Given intensive screening in which the reservation match quality is higher in period 1 than in period 2, the model currently predicts that a worker employed for two periods never cites match-quality reasons for turnover. If I further enrich the model such that each period a worker obtains a *noisy* private signal of the match, then the result will be that a worker employed for two periods possibly cites match-quality reasons but still mainly quotes exogenous reasons for separations.

#### 5.3 Employee selections and motivations

In this section, I consider the potential of an incentive-based theory to rationalize findings (esp. "interaction" results) described in Section 4, followed by discussing the interplay between selections and motivations.

## The potential of an incentive-based theory

As this article focuses on spot contracting prior to productions, pay-for-performance contracts cannot be used. One candidate is tournament models (Lazear and Rosen, 1981) where, due to the wage gap between a lower-level position and a higher-level position, the wage increase due to promotions serves as the prize to the winner of the promotion tournament and, in turn, gives rise to working incentives for multiple employees participating in the tournament. As opening up the competition for higher-level positions to external candidates limits the current employer's ability to use promotions for incentive purposes, Chan (1996) finds that the current employer handicaps external hiring and favors internal candidates for promotion. Another candidate is efficiency-wage models (Shapiro and Stiglitz, 1984), where wages are tied to job positions and the wage gap between a lower-level position and a higher-level position provides incentives for an employee. Similar to Chan (1996), Ke, Li, and Powell (2018) show that the current employer biases towards internal candidates when staffing higher-level positions, because hiring externally weakens the promotion prospect of insiders and therefore their incentives. Hence, tournament models and efficiency-wage models can rationalize one of "interaction" results in Section 4 that a worker employed for a while has a better promotion prospect than a newly employed worker.

Note, however, that the two candidate models above, as incentive-based theories, typically abstract away from worker heterogeneity and focus on one single firm, where there is worker entry at the bottom of the firm but no entry/exit at other levels. So, an incentive-based theory by itself (at least in its simple forms) is unsuitable for addressing several findings in this article (esp. "interaction" results concerning career dynamics within and across firms), which entails that a selection-based theory seems to be essential.

#### The interplay between selections and motivations

As a closing remark, I now consider the interplay between selections and motivations, where there is a (de)movational effect of utility of working on working incentives. For instance, one can enrich the above analysis such that each current employer faces a moral hazard problem, where each worker chooses an unobservable effort level  $e \in \{0, 1\}$  in each period. The cost of effort for a worker with ability level  $\theta$  and match quality  $\eta$  is  $c_{\theta}(0) = 0$  and

$$c_{\theta}(1) = \max\left\{0, C(\theta) - \omega\eta\right\},\$$

where  $C(\theta) > 0$ , with  $C'(\theta) < 0$ , captures the ability-based effort cost and  $\omega > 0$  characterizes the (de)movational effect of utility of working on incentives—a worker incurs a smaller (resp. larger) effort cost if his match  $\eta$  is higher (resp. lower) valued.<sup>23</sup> The productivity of a  $\theta \in \{0, 1\}$  ability worker who exerts effort  $e \in \{0, 1\}$  on position  $J \in \{L, M\}$  is given by

$$y_J(\theta; e) = (1 + \kappa_J e) y_J(\theta),$$

in which  $y_J(\theta)$  is as defined in Section 2 and  $\kappa_J > 0$  measures the importance of effort on position J's productivity (where there is no sabotage since  $\kappa_J > 0$ ).

Throughout the above analysis, I, however, abstract away from motivations or working incentives, and mainly focus on selections of employees, where as discussed above the novel aspect of this article is to incorporate selections regarding utility of working which has received very limited attentions in the literature. One justification for this focus is that in many real-world contexts, a worker's output is largely determined by his ability, i.e.,  $\kappa_J \rightarrow 0$  in the above specification. In turn, utility of working, though impacts on working incentives, has little or no effect on productivity. However, there exist many real-world situations where a (dis)satisfactory utility of working can (de)motivate incentives and, in turn, impact on productivity, i.e.,  $\kappa_J \rightarrow 0$ . So, the question is whether equilibrium results described in Section 4 will remain qualitatively unchanged when  $\kappa_J \rightarrow 0$ ?

Although I do not show it formally, my conjecture is that qualitative natures of this article's results are robust, whenever each newly employed worker has a strong incentive to prove to the current employer that he is high ability to receive a promotion and, in turn, signal to prospective employers that he is high ability.<sup>24</sup> The logic is that, due to utility of working which is the new aspect of this article, a current employer's promotion decision yields two types of returns to a worker: 1) a pecuniary return, i.e., in terms of the wage increase for a promoted employee; and 2) a non-pecuniary return, i.e., in terms of increased utility of working for a worker who intends to improve his match by entering a new firm.

Consider a newly employed worker experienced a low-quality match (i.e., he is unsatisfied with his current employer). If the worker cannot switch to a new firm, then a current employer's promotion yields a pecuniary return but not a non-pecuniary return. In this case, the worker does not exert effort whenever he is severely

 $<sup>^{23}</sup>$  This specification means that a worker's working incentives or effort cost is influenced both by his ability (as is true in a principal-agent model) and by his utility of working (which is the new aspect of the current setting).

<sup>&</sup>lt;sup>24</sup>This is analogous to career-concern models (Holmström, 1982) where employer learning is symmetric (i.e., the current employer and prospective employers can both observe the worker's performance). The difference is that, due to asymmetric employer learning, the only positive signal that a high ability worker can send to prospective employers is a promotion by his current employer. See Mukherjee (2008, 2010) and Bar-Isaac and Lévy (2022) for related analyses with asymmetric employer learning but without promotion signaling.

demotivated, i.e.,  $\omega > 0$  is large such that the effort cost outweighs the pecuniary return. This article, instead, studies a setting where a worker can change firms, which means that the current employer's promotion decision can advance the worker's future career at a new firm, then a current employer's promotion yields both a pecuniary return and a non-pecuniary return.<sup>25</sup> In turn, the worker's incentive can be potentially restored. That is, the worker exerts effort whenever he is not too demotivated, i.e.,  $\omega > 0$  is large but not too large such that the effort cost, though outweighs the pecuniary return, is less than the aggregation of pecuniary and non-pecuniary returns. As for a worker employed for a while, the demotivational effect on his productivity tends to be negligible. The logic is that, as the current employer's retention policy has already weeded out workers unsatisfied with their current employers, retained workers must be satisfied with their current employers and, in turn, associated with a negligible demotivational effect. Taken together, I conclude that equilibrium results should remain qualitatively unchanged so long as  $\omega > 0$  or the (de)movational effect of utility of working on working incentives is not too large.<sup>26</sup>

# 6 Conclusion

To wrap-up, utility of working, which captures a worker's idiosyncratic valuation for serving a firm, is clearly an important economic variable that influences a worker's career but has received very limited attentions in the literature. This article examines its role in driving labor-market dynamics. I hope to have shown that: 1) a partnership of utility of working—a taste-based, private-value, and time-varying match—and promotion signaling models à la Waldman (1984) can rationalize a wide range of labor-market phenomena concerning career dynamics; and 2) the new sequential-screening approach to study turnover provides a micro-foundation for the intra-firm employee retention policy, which is often treated as a black box in search-and-matching frameworks.

There are several directions where this article's analysis could be extended. First, as opposed to completely homogeneous firms, one can enrich the analysis with firm heterogeneity to derive new testable implications concerning career dynamics. Second, a worker's utility of working currently only depends on his own match. One can enrich this article's sequential-screening analysis with habit formation (e.g., the match becomes increasingly attractive when a worker accumulates firm tenure), network externality in the workplace

<sup>&</sup>lt;sup>25</sup>In line with the model, in the real world employees often "quit after getting a promotion" when they "really know ... [the job] is a bad fit," because a promotion "is seen as a sign an employee has a solid future at the company" and "can ... give a worker the confidence to seek a better job somewhere else." See https://www.wsj.com/lifestyle/careers/getting-promoted-often-leads-to-jumping-ship-new-data-reveal-cc95f1fc.

<sup>&</sup>lt;sup>26</sup>The above discussion focuses on utility of working's demotivational effect on incentives. Concerning its motivational effect, as a retained worker has a better match and is, in turn, less costly to be motivated/incentivized, one new aspect of the equilibrium will be that the current employer's promotion policy favors a worker employed for a while, while the current employer's rent-extraction motive is further strengthened.

(e.g., a worker's utility of working is partially determined by the average match quality of his coworkers),

etc. Third, as utility of working arguably affects a worker's career and drives labor-market dynamics, it

looks promising for future empirical research to test and estimate this economic variable.

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# A Appendix: Technical Details

# A.1 The simplified analysis

Given  $r \in (0, +\infty)$  and  $q \in (0, 1)$ , consider an optimization problem  $\phi_q(r) = \max_{\eta} [1 - (1 - q)G(\eta)] (r + \eta)$ . The first order condition is  $\Psi_q(\eta) = \frac{1 - (1 - q)G(\eta)}{(1 - q)g(\eta)} - \eta = r$ ,<sup>1</sup> where  $\Psi'_q(\eta) = \frac{1}{1 - q} \left( \frac{\partial \frac{1 - G(\eta)}{g(\eta)}}{\partial \eta} + q \frac{\partial \frac{G(\eta)}{g(\eta)}}{\partial \eta} \right) - 1 < 0$  because the log-concavity of  $G(\cdot)$  suggests  $\frac{\partial \frac{1 - G(\eta)}{g(\eta)}}{\partial \eta} \leq 0$  and  $\frac{\partial \frac{G(\eta)}{g(\eta)}}{\partial \eta} \leq 0$ . So,  $\Phi_q(r) = \Psi_q^{-1}(r)$  is well-defined. Below I study how  $\phi_q(r)$  and  $\Phi_q(r)$  change with r and q.<sup>2</sup>

**Lemma A1.**  $\phi_q(r)$  increases in r, and  $\Phi_q(r)$  decreases in r.

*Proof.* By the envelope theorem,  $\partial \phi_q(r)/\partial r = [1 - (1 - q)G(\Phi_q(r))] > 0$ . By the inverse function theorem,  $\Phi_q(r) = \Psi_q^{-1}(r)$  decreases in r since  $\Psi_q'(\eta) < 0$ .

**Lemma A2.**  $\phi_q(r)$  and  $\Phi_q(r)$  both increase in q.

*Proof.* By the envelope theorem,  $\partial \phi_q(r)/\partial q = G(\Phi_q(r))(r + \Phi_q(r)) > 0$ . Let q > q' and suppose, on the contrary, that  $\Phi_q(r) \leq \Phi_{q'}(r)$ .  $\Psi_q(\Phi_q(r)) > \Psi_{q'}(\Phi_{q'}(r))$  because

$$\frac{1 - (1 - q)G\left(\Phi_q(r)\right)}{(1 - q)g\left(\Phi_q(r)\right)} - \Phi_q(r) \ge \frac{1 - (1 - q)G\left(\Phi_{q'}(r)\right)}{(1 - q)g\left(\Phi_{q'}(r)\right)} - \Phi_{q'}(r) > \frac{1 - (1 - q')G\left(\Phi_{q'}(r)\right)}{(1 - q')g\left(\Phi_{q'}(r)\right)} - \Phi_{q'}(r),$$

where the first inequality employs the log-concavity of  $G(\cdot)$  and the second inequality uses q > q', which contradicts  $\Psi_q(\Phi_q(r)) = \Psi_{q'}(\Phi_{q'}(r)) = r$ . In turn,  $\Phi_q(r) > \Phi_{q'}(r)$  for  $\forall q > q'$ .

## One-stage screening for a worker employed in period 1

Whether or not the match is retained in period 2, the worker stays in period 2 with probability  $1 - G(\hat{\eta}_2^1)$ . The retention offer  $w_2^1 = e_2^1 - \hat{\eta}_2^1$  solves  $\max_{\hat{\eta}_2^1} \left[ 1 - G(\hat{\eta}_2^1) \right] \left[ y_2^1 - (e_2^1 - \hat{\eta}_2^1) \right] = \phi(y_2^1 - e_2^1)$ , which yields  $\hat{\eta}_2^1 = \Phi(y_2^1 - e_2^1)$ .

#### Sequential screening for a worker employed in period 0

There are two possible regimes: one with  $\hat{\eta}_1^1 \leq \hat{\eta}_2^2$  and the other with  $\hat{\eta}_1^1 > \hat{\eta}_2^2$ .

**Non-intensive screening.** When  $\hat{\eta}_1^1 \leq \hat{\eta}_2^2$ , the worker stays in period 2 with probability

$$q\frac{1-G(\hat{\eta}_2^2)}{1-G(\hat{\eta}_1^1)} + (1-q)\left[1-G(\hat{\eta}_2^2)\right] = \left(\frac{q}{1-G(\hat{\eta}_1^1)} + 1-q\right)\left[1-G(\hat{\eta}_2^2)\right].$$

The retention offer  $w_2^2 = e_2^2 - \hat{\eta}_2^2$  solves

$$\max_{\hat{\eta}_2^2} \left( \frac{q}{1 - G(\hat{\eta}_1^1)} + 1 - q \right) \left[ 1 - G(\hat{\eta}_2^2) \right] \left[ y_2^2 - (e_2^2 - \hat{\eta}_2^2) \right] = \left( \frac{q}{1 - G(\hat{\eta}_2)} + 1 - q \right) \phi(y_2^2 - e_2^2),$$

which yields  $\hat{\eta}_2^2 = \Phi(y_2^2 - e_2^2)$ . The expected payoff to stay in period 1 is

$$w_1^1 + \hat{\eta}_1^1 + \delta \left[ q e_2^2 + (1-q) \left( e_2^2 + \varphi \left( \hat{\eta}_2^2 \right) \right) \right] = w_1^1 + \hat{\eta}_1^1 + \delta \left[ e_2^2 + (1-q) \varphi \left( \hat{\eta}_2^2 \right) \right],$$

<sup>&</sup>lt;sup>1</sup>The solution to the optimization problem is given by the first-order condition, which is true when the distribution  $G(\cdot)$  has a large support.

<sup>&</sup>lt;sup>2</sup>For the limiting case where  $q \to 0$ , I express  $\phi_{q\to 0}(r)$  and  $\Phi_{q\to 0}(r)$ , respectively, as  $\phi(r)$  and  $\Phi(r)$ .

for a worker sampled match  $\hat{\eta}_1^1$  in period 1, where  $\varphi\left(\hat{\eta}_2^2\right) = \int_{\eta \ge \hat{\eta}_2^2} \eta - \hat{\eta}_2^2 dG(\eta)$ . So, the retention offer  $w_1^1 = e_1^1 - \hat{\eta}_1^1 - \delta\left[e_2^2 + (1-q)\varphi\left(\hat{\eta}_2^2\right)\right]$  solves

$$\begin{aligned} \max_{\hat{\eta}_{1}^{1}} \left[ 1 - G(\hat{\eta}_{1}^{1}) \right] \left\{ y_{1}^{1} - \left\{ e_{1}^{1} - \hat{\eta}_{1}^{1} - \delta \left[ e_{2}^{2} + (1 - q)\varphi\left(\hat{\eta}_{2}^{2}\right) \right] \right\} + \delta \left( \frac{q}{1 - G(\hat{\eta}_{2})} + 1 - q \right) \phi(y_{2}^{2} - e_{2}^{2}) \right\} \\ &= \left[ 1 - G(\hat{\eta}_{1}^{1}) \right] \left\{ y_{1}^{1} - \left( e_{1}^{1} - \delta e_{2}^{2} \right) + \delta(1 - q) \left[ \varphi\left(\hat{\eta}_{2}^{2}\right) + \phi(y_{2}^{2} - e_{2}^{2}) \right] + \hat{\eta}_{1}^{1} \right\} + \delta q \phi(y_{2}^{2} - e_{2}^{2}) \right] \end{aligned}$$

Plugging in  $\hat{\eta}_2^2 = \Phi(y_2^2 - e_2^2)$ , I obtain

$$\hat{\eta}_1^1 = \Phi\left(y_1^1 - \left(e_1^1 - \delta e_2^2\right) + \delta(1-q)\left[\varphi\left(\Phi(y_2^2 - e_2^2)\right) + \phi(y_2^2 - e_2^2)\right]\right).$$

**Intensive screening.** When  $\hat{\eta}_1^1 > \hat{\eta}_2^2$ , the worker stays in period 2 with probability

$$q + (1 - q) \left[ 1 - G(\hat{\eta}_2^2) \right] = 1 - (1 - q)G(\hat{\eta}_2^2).$$

The retention offer  $w_2^2 = e_2^2 - \hat{\eta}_2^2$  solves

$$\max_{\hat{\eta}_2^2} \left[ 1 - (1-q)G(\hat{\eta}_2^2) \right] \left[ y_2^2 - (e_2^2 - \hat{\eta}_2^2) \right] = \phi_q(y_2^2 - e_2^2)$$

which yields  $\hat{\eta}_2^2 = \Phi_q(y_2^2 - e_2^2)$ . The expected payoff to stay in period 1 is

$$w_1^1 + \hat{\eta}_1^1 + \delta \left[ q \left( e_2^2 - \hat{\eta}_2^2 + \hat{\eta}_1^1 \right) + (1 - q) \left( e_2^2 + \varphi \left( \hat{\eta}_2^2 \right) \right) \right] = w_1^1 + (1 + \delta q) \hat{\eta}_1^1 + \delta \left[ e_2^2 - q \hat{\eta}_2^2 + (1 - q) \varphi \left( \hat{\eta}_2^2 \right) \right],$$

for a worker sampled match  $\hat{\eta}_1^1$  in period 1. So, the retention offer  $w_1^1 = e_1^1 - (1+\delta q)\hat{\eta}_1^1 - \delta\left[e_2^2 - q\hat{\eta}_2^2 + (1-q)\varphi\left(\hat{\eta}_2^2\right)\right]$  solves

$$\begin{split} \max_{\hat{\eta}_{1}^{1}} \left[ 1 - G(\hat{\eta}_{1}^{1}) \right] \left\{ y_{1}^{1} - \left\{ e_{1}^{1} - (1 + \delta q) \hat{\eta}_{1}^{1} - \delta \left[ e_{2}^{2} - q \hat{\eta}_{2}^{2} + (1 - q) \varphi \left( \hat{\eta}_{2}^{2} \right) \right] \right\} + \delta \phi_{q} (y_{2}^{2} - e_{2}^{2}) \right\} \\ &= (1 + \delta q) \phi \left( \frac{y_{1}^{1} - \left( e_{1}^{1} - \delta e_{2}^{2} \right) + \delta \left[ (1 - q) \varphi \left( \hat{\eta}_{2}^{2} \right) + \phi_{q} (y_{2}^{2} - e_{2}^{2}) - q \hat{\eta}_{2}^{2} \right]}{1 + \delta q} \right). \end{split}$$

Plugging in  $\hat{\eta}_2^2 = \Phi_q(y_2^2 - e_2^2),$  I obtain

$$\hat{\eta}_1^1 = \Phi\left(\frac{y_1^1 - \left(e_1^1 - \delta e_2^2\right) + \delta\left[(1 - q)\varphi\left(\Phi_q(y_2^2 - e_2^2)\right) + \phi_q(y_2^2 - e_2^2) - q\Phi_q(y_2^2 - e_2^2)\right]}{1 + \delta q}\right).$$

Condition for intensive screening. Taken together,  $\hat{\eta}_1^1 > \hat{\eta}_2^2$  if and only if

$$y_1^1 - \left(e_1^1 - \delta e_2^2\right) + \delta(1 - q) \left[\varphi \left(\Phi(y_2^2 - e_2^2)\right) + \phi(y_2^2 - e_2^2)\right] < y_2^2 - e_2^2,\tag{1}$$

which I refer to as the condition for intensive screening.<sup>3</sup>

 $\frac{1}{3 \text{Denote } r_2^2 = y_2^2 - e_2^2. \quad \hat{\eta}_1^1 > \hat{\eta}_2^2 \text{ if } \frac{y_1^1 - (e_1^1 - \delta e_2^2) + \delta\left[(1 - q)\varphi(\Phi_q(r_2^2)) + \phi_q(r_2^2) - q\Phi_q(r_2^2)\right]}{1 + \delta q} < r_2^2 \text{ or } y_1^1 - (e_1^1 - \delta e_2^2) + \delta\left[(1 - q)\varphi(\Phi_q(r_2^2)) + \phi_q(r_2^2) - q\left(r_2^2 + \Phi_q(r_2^2)\right)\right] < r_2^2, \text{ which is implied by } y_1^1 - (e_1^1 - \delta e_2^2) + \delta(1 - q)\left[\varphi\left(\Phi(r_2^2)\right) + \phi(r_2^2)\right] < r_2^2,$ 

## A.2 The full analysis

Given a competitive market, the payoff to exit is equal to the expected payoff to switching to a new firm. For a worker employed in period 1, the period 2 payoff to exit is  $e_2^1 = v_2^1$  and the current employer's profit equals  $\phi(y_2^1 - v_2^1)$  in period 2. As for a worker employed in period 0, the payoff to exit is  $e_2^2 = v_2^2$  in period 2 and  $e_1^1 = v_1^1 + \delta \left[ v_2^1 + \varphi \left( \Phi(y_2^1 - v_2^1) \right) \right]$  in period 1, where  $v_2^1 + \varphi \left( \Phi(y_2^1 - v_2^1) \right)$  captures the period 2 expected payoff if the worker switched employers in period 1. Plugging them in (1), the condition for intensive screening given a competitive market is

$$y_1^1 - \left\{ v_1^1 + \delta \left[ v_2^1 + \varphi \left( \Phi(y_2^1 - v_2^1) \right) \right] - \delta v_2^2 \right\} + \delta(1 - q) \left[ \varphi \left( \Phi(y_2^2 - v_2^2) \right) + \phi(y_2^2 - v_2^2) \right] < y_2^2 - v_2^2.$$
(2)

When (2) holds, I obtain  $\hat{\eta}_2^2 = \Phi_q(y_2^2 - v_2^2)$  and

$$\hat{\eta}_{1}^{1} = \Phi \left( \left( 1 + \delta q \right)^{-1} \left\{ \begin{array}{c} y_{1}^{1} - \left( v_{1}^{1} + \delta \left[ v_{2}^{1} + \varphi \left( \Phi \left( y_{2}^{1} - v_{2}^{1} \right) \right) \right] - \delta v_{2}^{2} \right) \\ + \delta \left[ \left( 1 - q \right) \varphi \left( \Phi_{q} \left( y_{2}^{2} - v_{2}^{2} \right) \right) + \phi_{q} \left( y_{2}^{2} - v_{2}^{2} \right) - q \Phi_{q} \left( y_{2}^{2} - v_{2}^{2} \right) \right] \end{array} \right\} \right),$$
(3)

and the current employer's period 2 profit and period 1 profit are, respectively, given by  $\phi_q(y_2^2 - v_2^2)$  and

$$(1+\delta q)\phi\left((1+\delta q)^{-1}\left\{\begin{array}{c}y_1^1 - (v_1^1 + \delta\left[v_2^1 + \varphi\left(\Phi\left(y_2^1 - v_2^1\right)\right)\right] - \delta v_2^2\right)\\ +\delta\left[(1-q)\varphi\left(\Phi_q\left(y_2^2 - v_2^2\right)\right) + \phi_q\left(y_2^2 - e_2^2\right) - q\Phi_q\left(y_2^2 - v_2^2\right)\right]\end{array}\right\}\right).$$
(4)

**Proof of Lemma 1 and 2.** According to  $\phi(y_2^1 - v_2^1)$  and  $\phi_q(y_2^2 - v_2^2)$  described above, when analyzing a current employer's period 2 job assignments, I can focus on "output less poaching wage", i.e.,  $y_2^1 - v_2^1$  for a worker employed in period 1 and  $y_2^2 - v_2^2$  for a worker employed in period 0. As discussed in the main text, job assignments are fully efficient in period 2 because the output increase always outweight the wage increase, i.e.,  $(1 + \gamma)n_2(\alpha - 1) > n_2(\alpha - 1)$ .

**Proof of Lemma 3 and 4.** Given fully efficient job assignment in period 2, outputs and poaching wages satisfy  $y_2^1 = y_2^2$  and  $v_2^1 = v_2^2$ . In turn, (4) is equivalent to

$$(1+\delta q)\phi\left((1+\delta q)^{-1}\left\{y_{1}^{1}-v_{1}^{1}+\delta\left[\begin{array}{c}(1-q)\varphi\left(\Phi_{q}\left(y_{2}^{2}-v_{2}^{2}\right)\right)+\phi_{q}\left(y_{2}^{2}-e_{2}^{2}\right)\\-\varphi\left(\Phi\left(y_{2}^{1}-v_{1}^{1}\right)\right)-q\Phi_{q}\left(y_{2}^{2}-v_{2}^{2}\right)\end{array}\right]\right\}\right).$$

Thus, when studying a current employer's period 1 job assignments, I can focus on "output less poaching wage", i.e.,  $y_1^1 - v_1^1$  for a worker employed in period 0. Due to (A1), job assignments are not fully efficient in period 1. Below I pin down the unique threshold schooling level  $\hat{k}$ , and the promotion prospect of a high ability worker with schooling level above the threshold, i.e.,  $f(h;k) \in (0,1)$  for  $\forall k > \hat{k}$ .

Denote  $S(\theta; k, J)$  as the turnover rate of a  $\theta$  ability worker with schooling level k and assigned to position J. As a low ability worker has no promotion prospect in period 1, i.e., f(l; k) = 0, prospective employers update beliefs that a worker who switched employers after being assigned to the managerial position is high

because 
$$\partial \Phi_q(\cdot)/\partial q > 0$$
 and  $\varphi'(\cdot) < 0$  suggest  $\varphi\left(\Phi_q(r_2^2)\right) < \varphi\left(\Phi(r_2^2)\right)$  and  

$$\phi_q(r_2^2) - q\left(r_2^2 + \Phi_q(r_2^2)\right) = \underbrace{\left[1 - (1 - q)G\left(\Phi_q(r_2^2)\right)\right]\left(r_2^2 + \Phi_q(r_2^2)\right) - q\left(r_2^2 + \Phi_q(r_2^2)\right)}_{=(1 - q)\left[1 - G\left(\Phi_q(r_2^2)\right)\right]\left(r_2^2 + \Phi_q(r_2^2)\right)} < (1 - q)\phi(r_2^2).$$

ability with probability

$$\lambda(k,M) = \frac{p(h|k)f(h;k)S(h;k,M)}{p(h|k)f(h;k)S(h;k,M) + p(l|k)f(l;k)S(l;k,M)} = 1,$$

and a worker who switched employers after being assigned to the labor position is high ability with probability

$$\lambda(k,L) = \frac{p(h|k) \left[1 - f(h;k)\right] S(h;k,L)}{p(h|k) \left[1 - f(h;k)\right] S(h;k,L) + p(l|k) \left[1 - f(l;k)\right] S(l;k,L)} = \frac{1}{1 + \frac{p(l|k)}{p(h|k)} \frac{1}{1 - f(h;k)} \frac{S(l;k,L)}{S(h;k,L)}} < 1.$$

In turn,  $\lambda(k, M) = 1$  suggests that the poaching wage to a manager is  $n_1 \alpha + \delta \phi (\gamma n_2 \alpha)$ , while  $\lambda(k, L)$  yields that the poaching wage to a labor worker is

$$\max\{n_1, \lambda(k, L)n_1\alpha\} + \delta\left[\phi\left(\gamma n_2\right) + \lambda(k, L)\left(\phi\left(\gamma n_2\alpha\right) - \phi\left(\gamma n_2\right)\right)\right].$$
(5)

As  $f(h;k) \in (0,1)$  is a mixed strategy, the current employer is indifferent between assigning a high ability worker to the managerial position and to the labor position. Consequently, the poaching wage to a labor worker equals

$$n_1\alpha + \delta\phi(\gamma n_2\alpha) - (1+\gamma)n_1(\alpha - 1) = n_1(1+\gamma - \gamma\alpha) + \delta\phi(\gamma n_2\alpha).$$
(6)

Equating (5) and (6) yields that  $f(h;k) \in (0,1)$  can be uniquely pinned down by

$$\frac{1}{1+\frac{p(l|k)}{p(h|k)}\frac{1}{1-f(h;k)}\frac{S(l;k,L)}{S(h;k,L)}} = \frac{n_1\left(1+\gamma-\gamma\alpha\right)+\delta\left[\phi\left(\gamma n_2\alpha\right)-\phi(\gamma n_2)\right]}{n_1\alpha+\delta\left[\phi\left(\gamma n_2\alpha\right)-\phi(\gamma n_2)\right]},$$

where  $\frac{\partial}{\partial k} \left( \frac{p(l|k)}{p(h|k)} \right) < 0$  and  $\frac{\partial}{\partial k} \left( \frac{S(l;k,L)}{S(h;k,L)} \right) = 0.^4$  In turn, the promotion policy is biased towards more educated workers, i.e.,  $\partial f(h;k)/\partial k > 0$  for  $\forall k > \hat{k}$ , while the threshold schooling level  $\hat{k}$  is uniquely determined by

$$\hat{k} = \inf\left\{k\left|\frac{1}{1+\frac{p(l|k)}{p(h|k)}\frac{S(l;k,L)}{S(h;k,L)}} = \frac{n_1\left(1+\gamma-\gamma\alpha\right)+\delta\left[\phi\left(\gamma n_2\alpha\right)-\phi(\gamma n_2)\right]}{n_1\alpha+\delta\left[\phi\left(\gamma n_2\alpha\right)-\phi(\gamma n_2)\right]}\right\},\right.$$

where  $\frac{p(l|k=0)}{p(h|k=0)} = +\infty$  and  $\frac{p(l|k=\bar{k})}{p(h|k=\bar{k})}$  is sufficiently small.

As for the current employer's retention policy, given a competitive market, the period 2 "output less poaching wage" is  $(1+\gamma)n_2\alpha - n_2\alpha = \gamma n_2\alpha$  for a manager and  $(1+\gamma)n_2 - n_2 = \gamma n_2$  for a labor worker. The period 1 poaching wage equals  $n_1\alpha + \delta\phi(\gamma n_2\alpha)$  for a manager and is strictly greater than  $n_1 + \delta\phi(\gamma n_2)$  for a labor worker, so the period 1 "output less poaching wage" is  $(1+\gamma)n_1\alpha - (n_1\alpha + \delta\phi(\gamma n_2\alpha)) = \gamma n_1\alpha - \delta\phi(\gamma n_2\alpha)$ for a manager and is strictly lower than  $(1+\gamma)n_1 - (1+\delta\phi(\gamma n_2)) = \gamma n_1 - \delta\phi(\gamma n_2)$  for a labor worker. In turn, the condition for intensive screening given a competitive market in (2) holds for each worker, i.e.,

$$\gamma n_1 \alpha - \delta \phi(\gamma n_2 \alpha) + \delta \left[ (1 - q) \phi(\gamma n_2 \alpha) - q \varphi \left( \Phi(\gamma n_2 \alpha) \right) \right] < \gamma n_2 \alpha$$

for each high ability worker assigned to the managerial position,

$$\gamma n_1 - \delta \phi(\gamma n_2) + \delta \left[ (1 - q)\phi(\gamma n_2) - q\varphi\left(\Phi(\gamma n_2)\right) \right] < \gamma n_2,$$

<sup>&</sup>lt;sup>4</sup>As depicted in Figure 3, neither the poaching wage to a labor worker nor the poaching wage to a manager changes with schooling levels for  $\forall k > \hat{k}$ ; inserting these poaching wages into (3), turnover rates S(l; k, L), S(h; k, L), and S(h; k, M), where S(h; k, L) = S(h; k, M), are invariant to schooling levels for  $\forall k > \hat{k}$ .



Figure 3: Period 1 poaching wages and turnover rates.

for each low ability worker assigned to the labor position, which due to  $\frac{\partial r - \delta(1-q)\phi(r)}{\partial r} = 1 - \delta(1-q)\left[1 - G\left(\Phi(r)\right)\right] > 0$  implies

$$\gamma n_1 - \delta \phi(\gamma n_2) + \delta \left[ (1 - q)\phi(\gamma n_2 \alpha) - q\varphi\left(\Phi(\gamma n_2 \alpha)\right) \right] < \gamma n_2 \alpha,$$

for each high ability worker assigned to the labor position.

**Proof of Proposition 1.** In the analysis above, I obtain a unique equilibrium for each worker with schooling level above the threshold. For each worker with schooling level below the threshold, the promotion prospect is zero in period 1, i.e., f(h,k) = f(l,k) = 0 for  $\forall k \leq \hat{k}$ . In turn, establishing equilibrium uniqueness is analogous to that in Schönberg (2007), where the current employer does no make any job assignment. Specifically, due to f(h,k) = f(l,k) = 0 for  $\forall k \leq \hat{k}$ , prospective employers update beliefs that a worker who switched employers after being assigned to the labor position is high ability with probability

$$\lambda(k;L) = \frac{p(h|k)S(h;k,L)}{p(h|k)S(h;k,L) + p(l|k)S(l;k,L)}$$

For length considerations, I relegate the detail to the Web Appendix, where I establish a fixed point for the vector consisting of the retention offer and the poaching offer, i.e.,  $\{w_1^1(\theta; k, L), v_1^1(k, L)\}$  where  $\theta \in \{l, h\}$ . In turn, I can use  $w_1^1(\theta; k, L)$  and  $v_1^1(k, L)$  to pin down the turnover rate  $S(\theta; k, L)$ .

**Proof of Proposition 2.** Given intensive screening where  $\hat{\eta}_1^1 > \hat{\eta}_2^2$ , a worker employed in period 0 quits in period 2 for exogenous reasons with probability  $(1-q)G(\hat{\eta}_2^2)$  and for match-quality reasons with probability 0. In turn, he mainly cites exogenous reasons for turnover. As for a worker employed in period 1, he quits in period 2 for exogenous reasons with probability  $(1-q)G(\hat{\eta}_2^1)$  and for match-quality reasons with probability  $qG(\hat{\eta}_2^1)$ . When  $q \in (0.5, 1)$ , he primarily quotes match-quality reasons for turnover.

# Web Appendix (Not Intended for Publication)

In this document, I first provide omitted technical details. Second, I elaborate on the sequential-screening approach to study turnover. Third, I consider long-term contracts and discuss this article's relationships to dynamic mechanism designs and behavior-based price discrimination.

# 1 Omitted Details

To save notations, I drop (k, L) below because each high ability worker with schooling level below the threshold is assigned to the labor position. In period 1, the current employer's profit function is given by

$$\max_{w_1^1(\theta)} \left[ 1 - G\left(\hat{\eta}_1^1(\theta)\right) \right] \left[ y_1^1(\theta) - w_1^1(\theta) + \delta \phi_q \left( y_2^2(\theta) - v_2^2(\theta) \right) \right],$$

where

$$\begin{split} w_1^1(\theta) &= v_1^1 + \delta \left[ v_2^1(\theta) + \varphi \left( \Phi \left( y_2^1(\theta) - v_2^1(\theta) \right) \right) - v_2^2(\theta) \right] \\ &+ \delta \left[ q \Phi_q \left( y_2^2(\theta) - v_2^2(\theta) \right) - (1 - q) \varphi \left( \Phi_q \left( y_2^2(\theta) - v_2^2(\theta) \right) \right) \right] - (1 + \delta q) \hat{\eta}_1^1(\theta), \end{split}$$

which suggests

$$\hat{\eta}_{1}^{1}(\theta) = (1+\delta q)^{-1} \left\{ \begin{array}{c} v_{1}^{1} + \delta \left[ v_{2}^{1}(\theta) + \varphi \left( \Phi \left( y_{2}^{1}(\theta) - v_{2}^{1}(\theta) \right) \right) - v_{2}^{2}(\theta) \right] \\ + \delta \left[ q \Phi_{q} \left( y_{2}^{2}(\theta) - v_{2}^{2}(\theta) \right) - (1-q)\varphi \left( \Phi_{q} \left( y_{2}^{2}(\theta) - v_{2}^{2}(\theta) \right) \right) \right] - w_{1}^{1}(\theta) \end{array} \right\}.$$

Invoking the first order condition with respect to  $w_1^1(\theta)$  yields

$$w_1^1(\theta) = y_1^1(\theta) + \delta\phi_q \left( y_2^2(\theta) - v_2^2(\theta) \right) - (1 + \delta q) \frac{1 - G(\theta)}{g(\theta)},\tag{1}$$

where  $G(\theta) = G(\hat{\eta}_1^1(\theta))$  and  $g(\theta) = G'(\theta)$ . Totally differentiating (1) yields

$$\frac{dw_1^1(\theta)}{dv_1^1} = \frac{\left[g(\theta)^2 + (1 - G(\theta)) g'(\theta)\right] / g(\theta)^2}{1 + \left[g(\theta)^2 + (1 - G(\theta)) g'(\theta)\right] / g(\theta)^2} < 1,$$

where the inequality uses  $\left[g(\theta)^2 + (1 - G(\theta)) g'(\theta)\right] / g(\theta)^2 > 0$  because  $G(\cdot)$  is log-concave.

Due to f(h,k) = f(l,k) = 0 for  $\forall k \leq \hat{k}$ , prospective employers update beliefs that a worker who switched employers after being assigned to the labor position is high ability with probability  $\lambda = \frac{pG(h)}{pG(h)+(1-p)G(l)}$ where p = p(h|k) = 1 - p(l|k). In turn, the poaching offer equals

$$v_1^1 = \begin{cases} 1 + \delta \left[ \lambda \phi \left( \gamma \alpha \right) + (1 - \lambda) \phi(\gamma) \right] & \text{if } \lambda \le 1/\alpha, \\ \lambda \alpha + \delta \left[ \lambda \phi \left( \gamma \alpha \right) + (1 - \lambda) \phi(\gamma) \right] & \text{if } \lambda > 1/\alpha, \end{cases}$$

which is continuous in  $\lambda$  and, in turn, continuous in  $w_1^1(\theta)$ . Denote

$$V(l) = \begin{cases} 1 + \delta \phi(\gamma) & \text{if } \lambda \leq 1/\alpha, \\ \delta \phi(\gamma) & \text{if } \lambda > 1/\alpha, \end{cases} \quad \text{and} \quad V(h) = \begin{cases} 1 + \delta \phi(\gamma \alpha) & \text{if } \lambda \leq 1/\alpha, \\ \alpha + \delta \phi(\gamma \alpha) & \text{if } \lambda > 1/\alpha, \end{cases}$$

where V(h) > V(l) for  $\forall \lambda$ . I can rewrite the poaching offer as  $v_1^1 = \lambda V(h) + (1-\lambda)V(l)$ . Totally differentiating it yields

$$dv_{1}^{1} = \frac{p(1-p)\left(V(h) - V(l)\right)\left[g(l)G(l)dw_{1}^{1}(l) - g(h)G(h)dw_{1}^{1}(h)\right] / \left[pG(l) + (1-p)G(h)\right]^{2}}{1 + p(1-p)\left(V(h) - V(l)\right)\left[g(l)G(l)dw_{1}^{1}(l) - g(h)G(h)dw_{1}^{1}(h)\right] / \left[pG(l) + (1-p)G(h)\right]^{2}} < 1,$$

because V(h) > V(l) and  $G(\cdot)$  is log-concave.

Taken together, I find that 1)  $w_1^1(\theta)$  is continuous in  $v_1^1$ , and a one-unit increase in  $v_1^1$  increases  $w_1^1(\theta)$  by less than one unit; and 2)  $v_1^1$  is continuous in  $w_1^1(\theta)$ , and a one-unit increase in  $w_1^1(\theta)$  increases  $v_1^1$  by less than one unit. In turn, there is a fixed point for the vector  $\{w_1^1(\theta), v_1^1\}$  for each  $\theta \in \{l, h\}$ , while the turnover rate is given by  $S(\theta) = G(\hat{\eta}_1^1(\theta))$  where  $\hat{\eta}_1^1(\theta)$  is as defined above. Hence, the equilibrium is unique for each worker whose schooling level is below the threshold.

# 2 Sequential Screening

In this section, I further elaborate on the sequential-screening approach to study turnover.

# 2.1 Infinite periods

In the article, I focus on a three-period sequential screening problem, which is later integrated with promotion signaling models. I now study sequential screening with infinite number of periods.

Consider a worker employed in period t whose career spans for infinite number of periods. In period  $t+\tau$ , his output equals  $y_{t+\tau}^{\tau}$ , while his payoff to exit is  $e_{t+\tau}^{\tau}$ . Below, I derive a condition for intensive screening, where the reservation match quality decreases with firm tenure.

**Proposition 1.** If the value of "output less payoff to exit" rises with tenure, the reservation match quality decreases with tenure, i.e.,  $\partial y_{t+\tau}^{\tau} - \left(e_{t+\tau}^{\tau} - \delta e_{t+\tau+1}^{\tau+1}\right)/\partial \tau > 0$  for  $\forall \tau > 0$  implies  $\partial \hat{\eta}_{t+\tau}^{\tau}/\partial \tau < 0$  for  $\forall \tau > 0$ .

*Proof.* Suppose  $\partial \hat{\eta}_{t+\tau}^{\tau} / \partial \tau < 0$  for  $\forall \tau \geq k$ . The period t + k reservation match quality is given by

$$\hat{\eta}_{t+k}^{k} = \Phi\left(\frac{y_{t+k}^{k} - \left(e_{t+k}^{k} - \delta e_{t+k+1}^{k+1}\right) + \delta\left[(1-q)\varphi\left(\hat{\eta}_{t+k+1}^{k+1}\right) + \phi_q\left(\Psi_q\left(\hat{\eta}_{t+k+1}^{k+1}\right)\right) - q\hat{\eta}_{t+k+1}^{k+1}\right]}{1 + \delta q}\right).$$

If  $\hat{\eta}_{t+k-1}^{k-1} > \hat{\eta}_{t+k}^{k}$ , then the period t+k-1 reservation match quality is

$$\hat{\eta}_{t+k-1}^{k-1} = \Phi\left(\frac{y_{t+k-1}^{k-1} - \left(e_{t+k-1}^{k-1} - \delta e_{t+k}^{k}\right) + \delta\left[(1-q)\varphi\left(\hat{\eta}_{t+k}^{k}\right) + \phi_q\left(\Psi_q\left(\hat{\eta}_{t+k}^{k}\right)\right) - q\hat{\eta}_{t+k}^{k}\right]}{1 + \delta q}\right).$$

where  $\hat{\eta}_{t+k-1}^{k-1} > \hat{\eta}_{t+k}^k$  because  $y_{t+k-1}^{k-1} - \left(e_{t+k-1}^{k-1} - \delta e_{t+k}^k\right) < y_{t+k}^k - \left(e_{t+k}^k - e_{t+k+1}^{k+1}\right), \hat{\eta}_{t+k}^k > \hat{\eta}_{t+k+1}^{k+1}$ , and

$$(1-q)\underbrace{\varphi'(\hat{\eta})}_{<0} + \underbrace{\phi'_q(\Psi_q(\hat{\eta}))}_{>0}\underbrace{\Psi'_q(\hat{\eta})}_{<0} < 0.$$

Thus,  $\partial \hat{\eta}_{t+\tau}^{\tau} / \partial \tau < 0$  for  $\forall \tau > 0$  whenever  $\partial y_{t+\tau}^{\tau} - \left( e_{t+\tau}^{\tau} - \delta e_{t+\tau+1}^{\tau+1} \right) / \partial \tau > 0$  for  $\forall \tau > 0$ .

This result tells us that if a current employer's promotion policy becomes more efficient for more experienced workers, a current employer's retention policy corresponds to intensive screening, where the reservation

match quality decreases with firm tenure. The logic is analogous to that in the article. In turn, the model generates exit dynamics that the hazard rate of job ending declines with firm tenure.

## 2.2 Time-invariant matches

To further demonstrate the importance of a time-varying match, I modify the simplified model as follows. At the beginning of each period, a worker now either retains the previous period's match with probability  $q \in (0, 1]$ , or quits with probability 1 - q. In other words, the match is time-invariant, but each period there is an exogenous probability 1 - q that each worker separates with the employer.

 $\begin{array}{l} \textbf{Proposition 2. } \hat{\eta}_{1}^{1} = \hat{\eta}_{2}^{2} = \Phi\left(\frac{y_{1}^{1} - e_{1}^{1} + \delta e_{2}^{2} + \delta q\left(y_{2}^{2} - e_{2}^{2}\right)}{1 + \delta q}\right) \text{ if } y_{1}^{1} - \left(e_{1}^{1} - \delta e_{2}^{2}\right) \leq y_{2}^{2} - e_{2}^{2} \text{ (i.e., intensive screening)}, \\ and \ \Phi\left(y_{1}^{1} - e_{1}^{1} + \delta e_{2}^{2}\right) = \hat{\eta}_{1}^{1} < \hat{\eta}_{2}^{2} = \Phi(y_{2}^{2} - e_{2}^{2}) \text{ otherwise (i.e., non-intensive screening)}. \end{array}$ 

*Proof.* If  $\hat{\eta}_1^1 < \hat{\eta}_2^2$ ,  $w_2^2 = e_2^2 - \hat{\eta}_2^2$  solves  $\max_{\hat{\eta}_2^2} q \frac{1-G(\hat{\eta}_2^2)}{1-G(\hat{\eta}_1^1)} \left[ y_2^2 - (e_2^2 - \hat{\eta}_2^2) \right] = \frac{q\phi(y_2^2 - e_2^2)}{1-G(\hat{\eta}_1^1)}$ , which yields  $\hat{\eta}_2^2 = \Phi(y_2^2 - e_2^2)$ . The payoff to stay in period 1 is  $w_1^1 + \hat{\eta}_1^1 + \delta e_2^2$ , so  $w_1^1 = e_1^1 - \delta e_2^2 - \hat{\eta}_1^1$  solves

$$\max_{\hat{\eta}_1^1} \left[ 1 - G(\hat{\eta}_1^1) \right] \left[ y_1^1 - \left( e_1^1 - \delta e_2^2 - \eta_1 \right) + \delta \frac{q\phi(y_2^2 - e_2^2)}{1 - G(\hat{\eta}_1^1)} \right] = \left[ 1 - G(\hat{\eta}_1^1) \right] \left( y_1^1 - e_1^1 + \delta e_2^2 + \eta_1 \right) + \delta q\phi(y_2^2 - e_2^2),$$

which yields  $\hat{\eta}_{1}^{1} = \Phi (y_{1}^{1} - e_{1}^{1} + \delta e_{2}^{2}).$ 

If  $\hat{\eta}_1^1 \ge \hat{\eta}_2^2$ ,  $w_2^2 = e_2^2 - \hat{\eta}_1^1$  solve  $\max_{\hat{\eta}_2^2} q \left( y_2^2 - e_2^2 + \hat{\eta}_2^2 \right)$ , which yields  $\hat{\eta}_1^1 = \hat{\eta}_2^2$ . The payoff to stay in period 1 is  $w_1^1 + \hat{\eta}_1^1 + \delta e_2^2$ , so  $w_1^1 = e_1^1 - \delta e_2^2 - \hat{\eta}_1^1$  solves

$$\max_{\hat{\eta}_{1}^{1}} \left[ 1 - G(\hat{\eta}_{1}^{1}) \right] \left[ y_{1}^{1} - \left( e_{1}^{1} - \delta e_{2}^{2} - \hat{\eta}_{1}^{1} \right) + \delta q \left( y_{2}^{2} - e_{2}^{2} + \hat{\eta}_{1}^{1} \right) \right] = (1 + \delta q) \phi \left( \frac{y_{1}^{1} - e_{1}^{1} + \delta e_{2}^{2} + \delta q \left( y_{2}^{2} - e_{2}^{2} \right)}{1 + \delta q} \right),$$
which yields  $\hat{\eta}_{1}^{1} = \Phi \left( \frac{y_{1}^{1} - e_{1}^{1} + \delta e_{2}^{2} + \delta q \left( y_{2}^{2} - e_{2}^{2} \right)}{1 + \delta q} \right).$ 

The result tells us that when the match is time invariant, intensive screening where  $\hat{\eta}_1^1 = \hat{\eta}_2^2$  suggests that screening takes place in period 1 only. Following this logic, when there are more than three periods, the turnover rate is  $qG(\hat{\eta}_1^1) + 1 - q$  in period 1, and 1 - q in period 2 and onwards. In turn, the model yields exit dynamics that the hazard rate of job ending does not decline with tenure.<sup>1</sup>

# **3** Long-term Contracts

In the article, I focus on spot contracts, because long-terms contracts are time-inconsistent given a timevarying match. In this section, I consider long-term contracting or dynamic mechanisms, where the employer can commit to a wage schedule. I also consider a non-commitment conversion of the game, where the employer cannot commit and makes a wage offer in each period, i.e., a sequential-screening schedule.

## 3.1 Time consistency

For the ease of exposition, below I focus on equilibrium outcomes when T = 2. I find that dynamic contracts are time-consistent given fixed types, but time-inconsistent given shifting types.

<sup>&</sup>lt;sup>1</sup>The hazard rate of job ending declines with tenure only if the exogenous separation probability declines with tenure.

#### **Fixed types**

I first consider what happens given fixed types, i.e., q = 1.<sup>2</sup> As the period 1 type is always retained in period 2, absorbing exits imply that  $\hat{\eta}_1 \leq \hat{\eta}_2$  must hold for a dynamic mechanism. In turn, the period 1 objective function is

$$\max_{\hat{\eta}_1, \hat{\eta}_2} \int_{\hat{\eta}_1}^{\bar{\eta}} \left( \eta - \frac{1 - G(\eta)}{g(\eta)} + y_1 - e_1 + \delta e_2 \right) dG(\eta) + \delta \int_{\hat{\eta}_2}^{\bar{\eta}} \left( \eta - \frac{1 - G(\eta)}{g(\eta)} + y_2 - e_2 \right) dG(\eta).$$

If  $y_1 - e_1 + \delta e_2 > y_2 - e_2$ , then  $\Phi(y_1 - e_1 + \delta e_2) = \hat{\eta}_1 < \hat{\eta}_2 = \Phi(y_2 - e_2)$ ; otherwise,  $\hat{\eta}_1 = \hat{\eta}_2$  and, in turn, the above objective function reduces to

$$\max_{\hat{\eta}_1} \int_{\hat{\eta}_1}^{\bar{\eta}} \left( (1+\delta) \left( \eta - \frac{1 - G(\eta)}{g(\eta)} \right) + y_1 - e_1 + \delta e_2 + \delta \left( y_2 - e_2 \right) \right) dG(\eta),$$

from where I obtain  $\hat{\eta}_1 = \hat{\eta}_2 = \Phi\left(\frac{y_1 + \delta y_2 - e_1}{1 + \delta}\right)$ . As for sequential screening, when q = 1, Proposition 2 above suggests two possible regimes, which match

As for sequential screening, when q = 1, Proposition 2 above suggests two possible regimes, which match with the optimal mechanism described above. Hence, dynamic mechanisms are time consistent given fixed types. The logic follows from absorbing exits, which suggest that the reservation type monotonically increases over time, i.e.,  $\hat{\eta}_1 \leq \hat{\eta}_2$ . In turn, the employer has no incentive to price discriminate and deviate from its promised level  $\hat{\eta}_2$  in period 2.

#### Shifting types

I then examine what happens given shifting types, i.e.,  $q \in (0, 1)$ . Consider the extreme case where  $q \to 0$ , i.e., the period 1 type is always resampled in period 2. In this case, the optimal dynamic mechanism prescribes that the employer offers the highest possible wage in period 2, i.e.,  $w_2 = y_2$ , which suggests  $\hat{\eta}_2 = e_2 - y_2$ ; in turn, the period 1 objective function is

$$\max_{\hat{\eta}_1} \int_{\hat{\eta}_1}^{\bar{\eta}} \left( \eta - \frac{1 - G(\eta)}{g(\eta)} + y_1 - e_1 + \delta e_2 + \delta \int_{e_2 - y_2}^{\bar{\eta}} \left( \eta' + y_2 - e_2 \right) dG(\eta') \right) dG(\eta),$$

which yields  $\hat{\eta}_1 = \Phi\left(y_1 - e_1 + \delta e_2 + \delta \int_{e_2 - y_2}^{\bar{\eta}} (\eta' + y_2 - e_2) dG(\eta')\right)$ . Obviously, this mechanism is timeinconsistent because the employer has an incentive to renege on the highest possible wage in period 2. The logic follows from the fact that the period 1 type is known to be resampled in period 2, but there is no reporting of worker types in period 2. This sets contrast to sequential screening, where reporting of worker types (effectively) occurs in periods 1 and 2. Following this logic, I conclude that dynamic mechanisms are time-inconsistent when  $q \in (0, 1)$ .

## 3.2 Dynamic mechanism designs and behavior-based price discrimination

This article's sequential-screening analysis is closest to Tirole (2016), which studies a class of dynamic pricing games with absorbing exits, i.e., an agent cannot come back after terminating the relationship with the principal. Note that Tirole (2016) and this article are also closely related to the dynamic pricing literature with behavior-based price discrimination (BBPD) first studied in Chen (1997) and Fudenberg and Tirole (2000) (see Fudenberg and Villas-Boas, 2006 for a survey), but with an important difference. That is, due to

<sup>&</sup>lt;sup>2</sup>See the Appendix below for the detail on dynamic contracts.

absorbing exits, the clientele is positively selected over time in Tirole (2016) and this article, which is a *mirror image* of BBPD frameworks where the customer base is negatively selected over time.<sup>3</sup> Consequently, as established above, dynamic mechanisms are time-consistent given fixed types, which sets contrast to BBPD frameworks where dynamic mechanisms are typically time-inconsistent given fixed types.

Consider the simplest possible setting where a monopoly sells a non-durable good to a continuum of consumers. In this case, Coasian dynamics are obtained, i.e., high-valuation consumers first join the customer base followed by the entrance of low-valuation consumers. In other words, the customer base is *negatively* selected over time (i.e., a consumer who purchases later is associated a lower valuation of the product); in turn, the monopoly cannot commit to the monopolistic price for all periods because it has incentive to subsequently price discriminate and reduce prices to attract more low-valuation consumers. In contrast, Tirole considers a different setting, where a monopoly manages a clientele with absorbing exits, i.e., each consumer cannot come back in the future once he has exited the customer base. Given one single seller, each period's payoff to exit is equal to zero. In turn, on the contrary to Coasian dynamics, absorbing exits suggest that low-valuation consumers first quit the customer base followed by the exit of high-valuation consumers, i.e., the clientele is *positively* selected over time (i.e., a consumer who quits later is associated a higher valuation of the product). Consequently, the monopoly has no incentive to price discriminate, which suggests a commitment to the monopolistic price for all periods.

In this article, I apply Tirole's idea of absorbing exits to study a current employer's retention policy, but depart from Tirole (2016) and the BBPD literature in three important directions.

- 1. As opposed to the focus on a monopoly, I consider a competitive market, where the payoff to exit  $e_t$  in each period t is no longer equal to zero but endogenously determined.
- 2. As opposed to fixed types, I consider shifting types. In turn, dynamic mechanisms are time-inconsistent, so I focus on spot contracting, where a current employer can price discriminate based on an incumbent employee's mobility decision in the past.
- 3. I study a three-period framework.<sup>4</sup>

In summary, as detailed in the article, I show that absorbing exists, along with the above three departures, generate career dynamics consistent with a wide range of labor-market phenomena. In particular, if types are fixed over time, q = 1, i.e., the current employer's retention policy (i.e., sequential screening) is timeconsistent. The resulting equilibrium still captures internal-labor-market results in Corollary 1 and 2 as well as the external-labor-market result in Corollary 4 concerning the wage increase for switching firms; yet, as turnover arises in period 1 but not in period 2, the equilibrium does not match with exit dynamics in Corollary 3 and "interaction" results in Corollary 5 and 6.

<sup>&</sup>lt;sup>3</sup>Burdett and Coles (2003) and Stevens (2004) study the employer's wage schedule (with commitment) under the threat of incoming outside opportunities. These studies, however, employ on-the-job search frameworks (Burdett and Mortensen, 1998), where there is no information asymmetry at the contracting stage and thus no screening issue.

<sup>&</sup>lt;sup>4</sup>As noted in Tirole (2016), when there are more than two periods, solving for equilibrium outcomes given non-commitment is notoriously difficult because a dynamic pricing problem cannot be boiled down to an optimization problem.

## Appendix: Dynamic mechanisms with fixed types

Consider a dynamic mechanism design problem with fixed types, where each worker draws a private-value and time-invariant value  $\eta \stackrel{d}{\sim} G(\cdot)$  in period 0. In each period  $t \in \{1, \dots, T\}$  where  $T < \infty$ , a worker produces output  $y_t$  if he stays with the employer, and obtains a payoff to exit  $e_t$  if he splits from the employer. Exits are absorbing, i.e., a worker cannot be re-employed after terminating the employment relationship.

## Preliminaries

By the revelation principle, I can focus on direct mechanisms,  $\mathcal{M} : \eta \mapsto \{\mathbf{w}(\eta), \mathbf{x}(\eta)\}$ , where each worker reports his type at the beginning of period 1 and the mechanism specifies for each worker type  $\eta$  a sequence of wages  $\mathbf{w}(\eta) = \{w_t(\eta)\}_{t=1}^T$  and a sequence of labor supply  $\mathbf{x}(\eta) = \{x_t(\eta)\}_{t=1}^T$ . As each worker's labor supply equals either zero or one in each period, the period t labor supply of a type  $\eta$  worker is described by a stochastic variable  $x_t(\eta) \in [0, 1]$ . Without loss of generality, I can focus on deterministic contracts where  $x_t(\eta) \in \{0, 1\}$ . Given absorbing exits,  $x_t(\eta) = 0$  suggests  $x_s(\eta) = 0$  for  $\forall s > t$ . Then, the period t labor supply of a type  $\eta$  worker is  $X_t(\eta) = \prod_{s=1}^t x_s(\eta)$ , and the discounted aggregate labor supply is  $\sum_{t=1}^T \delta^{t-1} X_t(\eta)$ . In turn, the inter-temporal payoff to a type  $\eta$  worker is

$$U_t(\eta) = \sum_{s=t}^T \delta^{s-t} X_s(\eta) \left[ \eta + w_s(\eta) \right],$$

while the inter-temporal payoff to an employer is

$$V_t = \int_{\underline{\eta}}^{\overline{\eta}} \left\{ \sum_{s=t}^T \delta^{s-t} X_s(\eta) \left[ y_s - w_s(\eta) \right] \right\} dG(\eta).$$

Dynamic mechanisms must satisfy an incentive-compatible constraint, i.e.,

$$U_1(\eta) \equiv U_1(\eta; \eta) \ge U_1(\eta; \eta'), \forall \eta' \neq \eta$$
(IC)

where  $U_1(\eta; \eta') = \sum_{t=1}^T \delta^{t-1} X_t(\eta') [\eta + w_t(\eta')]$  is a type  $\eta$  worker's inter-temporal payoff when reporting as type  $\eta'$ . It is standard to show (IC) is equivalent to a monotonic allocation, i.e., the discounted aggregate labor supply  $\frac{dU_1(\eta)}{d\eta} = \sum_{t=1}^T \delta^{t-1} X_t(\eta)$  increases with  $\eta$ . Dynamic mechanisms must also satisfy an individualrationality constraint, i.e.,

$$U_t(\eta) \ge e_t, \forall \eta \tag{IR-}t)$$

where  $e_t$  is the period t payoff to exit, a.k.a. reservation payoff. The optimal mechanism solves

$$\sup_{\{\mathbf{w}(\eta),\mathbf{x}(\eta)\}} V_1 \quad \text{s.t.} \quad (\text{IC}) \text{ and } (\text{IR-}t), \forall t > 0.$$

Using integration by parts, I obtain

$$\begin{split} \int_{\underline{\eta}}^{\overline{\eta}} U_t(\eta) dG(\eta) &= U_t(\eta) G(\eta) \big|_{\underline{\eta}}^{\overline{\eta}} - \int_{\underline{\eta}}^{\overline{\eta}} G(\eta) dU_t(\eta) = U_t(\overline{\eta}) - \int_{\underline{\eta}}^{\overline{\eta}} \left\{ \frac{dU_t(\eta)}{d\eta} G(\eta) \right\} d\eta \\ &= U_t(\underline{\eta}) + \int_{\underline{\eta}}^{\overline{\eta}} \left\{ \frac{dU_t(\eta)}{d\eta} \left[ 1 - G(\eta) \right] \right\} d\eta = e_t + \int_{\underline{\eta}}^{\overline{\eta}} \left\{ \sum_{s=t}^T \delta^{s-t} X_s(\eta) \left[ 1 - G(\eta) \right] \right\} d\eta, \end{split}$$

which yields a recursive formula

$$\begin{aligned} \mathbf{E}_{\eta} \left[ U_{t}(\eta) \right] &= e_{t} + \int_{\underline{\eta}}^{\bar{\eta}} X_{t}(\eta) \left[ 1 - G(\eta) \right] d\eta + \delta \left( \mathbf{E} \left[ U_{t+1}(\eta) \right] - e_{t+1} \right) \\ &= e_{t} - \delta e_{t+1} + \int_{\underline{\eta}}^{\bar{\eta}} X_{t}(\eta) \left[ 1 - G(\eta) \right] d\eta + \delta \mathbf{E} \left[ U_{t+1}(\eta) \right]. \end{aligned}$$

In turn, I obtain

$$\mathbf{E}_{\eta} \left[ U_t(\eta) \right] = \sum_{s=t}^{T} \delta^{s-t} \left( e_s - \delta e_{s+1} \right) + \int_{\underline{\eta}}^{\overline{\eta}} \left\{ \sum_{s=t}^{T} \delta^{s-t} X_s(\eta) \left[ 1 - G(\eta) \right] \right\} d\eta,$$

which can be decomposed to a value outside the relationship and a value within the relationship, i.e.,

$$\int_{\underline{\eta}}^{\overline{\eta}} \left\{ \sum_{s=t}^{T} \delta^{s-t} \left( 1 - X_s(\eta) \right) \left( e_s - \delta e_{s+1} \right) \right\} dG(\eta) + \int_{\underline{\eta}}^{\overline{\eta}} \left\{ \sum_{s=t}^{T} \delta^{s-t} X_s(\eta) \left( \frac{1 - G(\eta)}{g(\eta)} + e_s - \delta e_{s+1} \right) \right\} dG(\eta),$$

where  $\frac{1-G(\eta)}{g(\eta)}$  captures the information rent for a type  $\eta$  worker. In turn, an employer's inter-temporal payoff can be rewritten as the difference of the expected surplus and the worker's expected payoff within the relationship, i.e.,

$$V_1 = \int_{\underline{\eta}}^{\overline{\eta}} \left\{ \sum_{t=1}^T \delta^{t-1} X_t(\eta) \left[ y_t + \eta - \left( \frac{1 - G(\eta)}{g(\eta)} + e_t - \delta e_{t+1} \right) \right] \right\} dG(\eta),$$

where  $\Gamma(\eta) = \eta - \frac{1 - G(\eta)}{G(\eta)}$  is the standard virtual surplus which increases in  $\eta$ .

#### The optimal mechanism

The optimal mechanism  $\{\mathbf{w}(\eta), \mathbf{x}(\eta)\}$  maximizes  $V_1$  described above subject to (IC). Define  $\hat{\eta}_t$  as the reservation type for period t, i.e.,

$$\hat{\eta}_t = \inf\left\{\eta \left|\sum_{s=t}^T \delta^{s-t} X_s(\eta) > 0\right.\right\},\,$$

which corresponds to the lowest type with a non-zero labor supply for each period t onward. As (IC) means that  $\sum_{t=1}^{T} \delta^{t-1} X_t(\eta)$  increases with  $\eta$ , given absorbing exits, i.e.,  $x_t(\eta) = 0$  implies  $x_s(\eta) = 0$  for  $\forall s > t$ , the reservation type monotonically increases over time, i.e.,  $\hat{\eta}_{t+1} \ge \hat{\eta}_t$  for  $\forall t > 0$ . Then, the labor supply schedule  $\{x_t(\eta)\}_{t=1}^T$  can be transformed to a series of reservation types  $\{\hat{\eta}_t\}_{t=1}^T$ . In turn, the optimal mechanism is determined by an optimization problem, i.e.,

$$\sup_{\{\hat{\eta}_t\}_{t=1}^T} \sum_{t=1}^T \delta^{t-1} \int_{\hat{\eta}_t}^{\bar{\eta}} \left\{ \eta - \frac{1 - G(\eta)}{g(\eta)} + y_t - e_t + \delta e_{t+1} \right\} dG(\eta) \quad \text{s.t.} \quad \hat{\eta}_{t+1} \ge \hat{\eta}_t, \forall t > 0.$$

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